

# **Hopf algebras in Noncommutative Geometry and Physics**

**Proceedings of the Brussels conference  
Koninklijke Vlaamse Academie van België  
voor Wetenschappen en Kunsten  
May 28-June 1, 2002**

Edited by

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## Preface

This volume contains the proceedings of meeting “Hopf algebras and quantum groups”, which was held from May 28 until June 1, 2002, at the *Koninklijke Vlaamse Academie van België voor Wetenschappen en Kunsten*, the Royal Academy located in the historic centre of Brussels. The conference focussed at new results on classical Hopf algebras, with emphasis on the classification theory of finite dimensional Hopf algebras, categorical aspects of Hopf algebras, connections with Mathematical Physics, and the recent developments in the theory of corings and of quasi-Hopf algebras.

The meeting was supported by: the project “Noncommutative Geometry” (NOG) of the European Science Foundation (ESF); the bilateral project “New computational, geometric and algebraic methods applied to quantum groups and differential operators” of the Flemish and Chinese governments; the bilateral project “Hopf algebras in Algebra, Topology, Geometry and Physics” of the Flemish and Romanian governments; the Flemish Science Foundation FWO-Vlaanderen; the Royal Flemish Academy of Belgium (KVAB); the Free University of Brussels (VUB); the TMR project “Algebraic Lie Representations”. We wish to thank the ESF, the Flemish government, the FWO-Vlaanderen, the KVAB, VUB and TMR for enabling us to organize this meeting.

Our thanks also go to the participants, the speakers and the authors. J.-P. Tignol kindly gave us the permission to use his “Fesart style file”, used previously in volume 208 of the Lecture Notes in Pure and Applied Mathematics Series.

All papers have been individually refereed, and we thank the anonymous referees.

Stefaan Caenepeel  
Freddy Van Oystaeyen  
December 2003

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# Program

## Tuesday, May 28

- 10.00-10.50 B. Pareigis (München)  
*On Symbolic Computations with Elements in Braided Monoidal Categories*
- 10.50-11.20 Coffee
- 11.20-11.55 L. Kadison (Göteborg)  
*Antipodes at Depth Two*
- 12.00-12.35 A. Solotar (Buenos Aires)  
*Hochschild homology of trivial extensions*
- 12.35-14.20 Lunch
- 14.20-15.10 T. Brzeziński (Wales at Swansea)  
*Why corings?*
- 15.15-15.50 Y. Sommerhäuser (München)  
*Self-dual modules of semisimple Hopf algebras*
- 15.50-16.20 Coffee
- 16.20-16.55 M. Farinati (Buenos Aires)  
*Hochschild homology of Generalized Weyl Algebras*
- 17.00-17.35 S. Natale (Córdoba)  
*On the semi-solvability of semisimple Hopf algebras of small dimension*
- 17.45-19.00 Reception in the Marble Room

## Wednesday, May 29

- 09.00-09.50 H.-J. Schneider (München)  
*On the classification of pointed Hopf algebras*
- 09.55-10.30 R. Wisbauer (Düsseldorf)  
*Galois corings*
- 10.30-11.00 Coffee
- 11.00-11.35 J. Gomez Torrecillas (Granada)  
*Semisimple corings*
- 11.40-12.15 S. Caenepeel (Brussels)  
*Cleft entwining structures and Morita contexts associated to a coring*
- 12.15-14.00 Lunch
- 14.00-14.35 P. Schauenburgh (München)  
*Some properties of finite quasi-Hopf algebras*
- 14.40-15.15 T. Lenagan (Edinburgh)  
*Coinvariants and co-orbits for quantum matrices*
- 15.20-15.55 P. Hajac (München)  
*Fredholm Index and locally trivial noncommutative Hopf fibration*
- 16.00-16.30 Coffee

## Thursday, May 30

- 09.30-10.20 S. Montgomery (Univ. of Southern California)  
*Properties of algebras invariant under twisting*
- 10.20-10.50 Coffee
- 10.50-11.25 C. Menini (Ferrara)  
*Quantum groups of dimension 16 with the Chevalley property*
- 11.30-12.05 D. Bulacu (Bucharest)  
*Hopf modules for quasi-Hopf algebras*
- 12.05-14.00 Lunch break
- 14.00-14.50 K. Szlachányi (Budapest)  
*Adjointable lax monoidal functors and bialgebroids*
- 14.55-15.30 G. Böhm (Budapest)  
*Hopf algebroids with bijective antipode*
- 15.30-16.00 Coffee
- 16.00-16.35 J. Cuadra (Almería and Antwerp)  
*The Brauer group of the Dihedral group respect to a quasi-triangular structure*
- 16.40-17.15 Y. Zhang (Antwerp and Fiji)  
*The equivariant Brauer group of an infinite group*
- 17.20-17.55 D. Wang (Brussels and Qufu Univ.)  
*Twistings, crossed coproducts and Hopf Galois coextensions*
- 20.00 Congress Dinner

## Friday, May 31

- 09.30-10.20 S. Majid (Queen Mary College, London)  
*Dirac operators and Riemannian geometry on Hopf algebras*
- 10.20-10.50 Coffee
- 10.50-11.25 W. Chin (De Paul Univ., Chicago)  
*Prime Spectra of Quantized Hyperalgebras*
- 11.30-12.05 C. Cibils (Montpellier)  
*Hochschild cohomology of Hopf algebras*
- 12.05-14.00 Lunch break
- 14.00-14.35 M. Beattie (Mount Allison University)  
*Hopf algebras of dimension 14*
- 14.40-15.15 G. Militaru (Bucharest)
- 15.15-15.45 Coffee

- 15.45-16.20 F. Panaite (Bucharest)  
*Hopf bimodules as modules over a diagonal crossed product algebra*
- 16.25-17.00 X. Gomez (Queen Mary College, London)  
*Relating Woronowicz's quantum Lie algebras and Majid's braided Lie algebras*
- 17.05-17.40 P. Wisniewski (Torun)  
*The Lasker-Noether theorem for commutative and noetherian module algebras over a pointed Hopf algebra*

**Saturday, June 1**

- 09.30-10.20 D. Sternheimer and Ph. Bonneau (Univ. de Bourgogne)  
*Topological Hopf algebras, quantum groups and deformation quantization*
- 10.20-10.50 Coffee
- 10.50-11.25 C. Frønsdahl (UCLA)
- 11.30-12.05 A. Paolucci (Leeds)  
*Cuntz Algebras and Dynamical Quantum Group  $SL(2)$*
- 12.10-12.40 C. Ohn (Reims)  
*Quantum  $SL(3)$ 's: the missing case*