CORRECTION TO "FROBENIUS FUNCTORS OF THE SECOND KIND"

S. CAENEPEEL, E. DE GROOT, AND G. MILITARU

Let k be a commutative ring, and H a finitely generated projective Hopf algebra over k. It follows from the Fundamental Theorem for Hopf algebras that the integral space of H is finitely generated and projective of rank one. It is stated in [1, Cor. 3.5] that the integral spaces $\int_H^l$ in H and $\int_{H^*}^l$ on H are isomorphic. This is not true: $\int_{H^*}^l$ is isomorphic to the dual of $\int_H^l$. The mistake is caused by an inaccuracy in the preceding Theorem 3.4. We present the correct versions, and give an alternative proof of Corollary 3.5.

Theorem 3.4 Let H be Hopf algebra over a commutative ring k and I a projective k-module of rank one. Then the following assertions are equivalent.

i) H/k is I-Frobenius;
ii) H is finitely generated and projective and H^*/k is I^*-Frobenius;
iii) H is finitely generated and projective and $\int_H^l \cong I^*$;
iv) H is finitely generated and projective and $\int_H^r \cong I^*$;
v) H is finitely generated and projective and $\int_{H^*}^l \cong I$;
vi) H is finitely generated and projective and $\int_{H^*}^r \cong I$.

Proof. <u>v) $\Rightarrow$ ii)</u>. It follows from the Fundamental Theorem for Hopf algebras that we have an isomorphism of left H^*-modules (cf. [1, Theorem 3.2])

$$\alpha : \int_{H^*}^l \otimes H = I \otimes H \to H^*$$

This induces an isomorphism

$$\beta : \ H \to H^* \otimes I^*$$

β is left H^*-linear, so it follows from ii) of [1, Theorem 3.1] (with H replaced by H^*) that H^*/k is I^*-Frobenius.
The proof of the other implications is presented correctly in [1]. □

1991 *Mathematics Subject Classification.* 16W30.
Research supported by the bilateral project "Hopf Algebras in Algebra, Topology, Geometry and Physics" of the Flemish and Romanian governments.

Corollary 3.5 Let H be a finitely generated projective Hopf algebra over a commutative ring k. Then we have isomorphisms of k-modules

$$\int_{H^*}^{l} \cong \int_{H^*}^{r} \cong \left(\int_{H}^{l}\right)^* \cong \left(\int_{H}^{r}\right)^*.$$

Proof. This follows immediately from Theorem 3.4. Let us present an alternative proof, taken from the forthcoming [2]. Let $I = \int_{H^*}^{l}$ and $J = \int_{H}^{l}$ be the spaces of left integrals on and in H. Consider the isomorphism

$$\alpha :\ I \otimes H \to H^*,\ \alpha(\varphi \otimes h) = h \cdot \varphi,$$

with $\langle \varphi \cdot h, k\rangle = \langle \varphi, kh\rangle$, coming from the Fundamental Theorem. If $t \in J$, then

$$\alpha(\varphi \otimes t)(h) = \langle \varphi, ht\rangle = \langle \varphi, t\rangle\varepsilon(h),$$

so α restricts to a monomorphism $\tilde{\alpha} :\ I \otimes J \to k\varepsilon$. If I and J are free of rank one, then $\tilde{\alpha}$ is an isomorphism, as there exist $\varphi \in I$ and $t \in J$ such that $\langle \varphi, t\rangle = 1$ (see for example [3, Theorem 31], [5]. Hence $\tilde{\alpha}$ is an isomorphism after we localize at a prime ideal p of k, and this implies that $\tilde{\alpha}$ is itself an isomorphism. Consequently $J^* \cong I$. □

The referee pointed out to us that the correct version of Corollary 3.5 is already stated in [4, Remark 4.2], and is attributed to the first author. Probably this resulted from discussions to Lars Kadison's visit to the Vrije Universiteit Brussel in the fall of 2000.

References

[1] Caenepeel, S.; De Groot, E.; Militaru, G. Frobenius functors of the second kind. Comm. Algebra **2002** (11), 30, 5359–5391.

[2] Caenepeel, S.; Guédénon, T. Fully bounded noetherian rings and Frobenius extensions, preprint 2005.

[3] Caenepeel, S.; Militaru, G.; Zhu, Shenglin "Frobenius and separable functors for generalized module categories and nonlinear equations", Lect. Notes in Math. **1787**, Springer Verlag, Berlin, 2002.

[4] Kadison, L.; Stolin, A.A. An approach to Hopf algebras via Frobenius coordinates II. J. Pure Appl. Algebra **2002** (176), 127–152.

[5] Pareigis, B. When Hopf algebras are Frobenius Algebras. J. Algebra **1971** (18), 588–596.

Faculty of Engineering Sciences, Vrije Universiteit Brussel, VUB, B-1050 Brussels, Belgium
E-mail address: `scaenepe@vub.ac.be`
URL: `http://homepages.vub.ac.be/~scaenepe/`

Faculty of Engineering Sciences, Vrije Universiteit Brussel, VUB, B-1050 Brussels, Belgium
E-mail address: `scaenepe@vub.ac.be`

Faculty of Mathematics, University of Bucharest, Str. Academiei 14, RO-70109 Bucharest 1, Romania
E-mail address: `gmilit@al.math.unibuc.ro`