Pattern formation without diffraction matching in optical parametric oscillators with a metamaterial

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Abstract: We consider a degenerate optical parametric oscillator containing a left-handed material. We show that the inclusion of a left-handed material layer allows for controlling the strength and sign of the diffraction coefficient at either the pump or the signal frequency. Subsequently, we demonstrate the existence of stable dissipative structures without diffraction matching, i.e., without the usual relationship between the diffraction coefficients of the signal and pump fields. Finally, we investigate the size scaling of these light structures with decreasing diffraction strength.

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References and links
1. Introduction

Frequency conversion by means of quadratic media in degenerate optical parametric oscillators (DOPO) is a fundamental technique for the generation of tunable coherent radiation [1, 2]. When used in broad area devices, the coupling between diffraction and nonlinearity can induce modulational instability of the homogeneous output beam, leading to the formation of dissipative periodic structures [3]. Besides spatially periodic structures, DOPOs also support localized structures. In the study of these structures, three different regimes may be distinguished depending on the magnitude of $k_M$, the most unstable wavenumber at the onset of modulational instability. (i) If $k_M$ is finite, modulational instability appears subcritically and there exists a
pinning domain where localized structures are stable [4, 5]. (ii) If \( k_M \) is small (long-wavelength regime), the modulational instability occurs close to the limit point associated with a domain of bistability. The long-wavelength pattern formation process is altered and leads to the formation of localized structures [6]. (iii) If \( k_M \) vanishes, the homogeneous steady states are modulationally stable. In this case, the stabilization of localized structures requires bistability between two homogeneous steady states; the resulting structures are often called phase solitons or domain walls [7–12]. The latter two types of spatial confinement of light (cases (ii) and (iii)) are generated from noise and the late-time kinetics of their formation obeys a power law [6, 12]. An overview of this subject can be found in Refs. [13–17].

In a different subfield of optics, scientists have recently established metamaterials with negative permittivity and permeability, often called left-handed materials (LHM) [18, 19]. Such materials have been first fabricated at microwave frequencies [20, 21], but are now also realized in the optical domain [22, 23]. Metamaterials are shown to exhibit novel electromagnetic phenomena [18, 24] and can be used in nonlinear optical devices [25–29]. In particular, the formation of dissipative structures in a Kerr resonator containing a LHM has been studied in Refs. [30–32], where it was shown that the addition of a layer of LHM strongly alters the spatiotemporal dynamics and provides the ability of diffraction management.

In this paper, we apply the technique of diffraction management to degenerate optical parametric oscillators containing a LHM layer in their cavity. We show that this device can operate without diffraction matching, i.e., without the usual relationship between the diffraction coefficients of signal and pump fields. The technique allows to control the magnitude and the sign of the diffraction coefficient of the signal and pump fields. This can be achieved by tuning the thickness of the left-handed layer and of the quadratic crystal. Finally, we present a study of the size scaling of the emerging diffractive patterns.

2. Mean field model, diffraction management and steady-state solutions

We consider a ring cavity containing two material layers and driven by a coherent input beam at angular frequency \( \omega \) (see Fig. 1). The first layer contains a material with a quadratic nonlinearity, coupling the pump wave (\( \omega \)) to the signal wave (\( \omega/2 \)). Both waves are phase matched in this layer to ensure high conversion efficiency. The second layer consists of a LHM. We assume that the LHM has a linear optical response, so that there is no need for phase matching in this layer; the nonlinearities in the system are only due to the presence of the \( \chi^{(2)} \) medium. In the currently existing metamaterials, the left-handedness is limited to a very small frequency band due to the resonant coupling with electromagnetic radiation. Therefore, only the case where one of both optical fields experiences a negative index of refraction (\( n_s < 0 \)) and the index of the pump field positive (\( n_p > 0 \)).

![Fig. 1. Schematic setup of the degenerate optical parametric oscillator with a LHM.](image)

Here, we consider type I parametric oscillation, where polarization effects are unimportant.
The propagation of light in the quadratic crystal can be described by the reduced Maxwell’s equations. Using the technique from Ref. [30], we have derived the following propagation equations for the pump and signal amplitudes $A_s$ and $A_p$, which are valid in both layers (with $\chi^{(2)} = 0$ in the LHM, since we assume that this layer is linear):

$$\frac{\partial A_s}{\partial \tau} + \frac{n_s}{c} \frac{\partial A_s}{\partial \xi} = \frac{i \omega \chi^{(2)}}{c \eta_h} A_p A_s^* + i \frac{c}{2 \omega n_s} \nabla_{\perp}^2 A_s,$$

(1)

$$\frac{\partial A_p}{\partial \tau} + \frac{n_p}{c} \frac{\partial A_p}{\partial \xi} = i \frac{\omega \chi^{(2)}}{2 c \eta_p} A_s^2 + i \frac{c}{\omega n_p} \nabla_{\perp}^2 A_p,$$

(2)

We thus find that the propagation equations keep the same form in LHMs, but that the difference between the index of refraction $n_{s,p}$ and the characteristic impedance $\eta_{s,p}$ in such media must be carefully taken into account. Since $n_s < 0$, Eqs. (1)-(2) show that the diffraction of the signal beam acts with a negative sign in the LHM.

The signal wave propagates first through the quadratic crystal with positive index and then through the left-handed layer with negative index. This means that the diffraction in both layers will counteract and partly compensate. This property is reflected in the mean-field model that we have derived in earlier work from Eqs. (1)-(2) with appropriate boundary conditions at the mirrors [32]:

$$\frac{\partial A_s}{\partial \tau} = - (1 + i \Delta_s) A_s + A_p A_s^* + i D_s \nabla_{\perp}^2 A_s,$$

(3)

$$\frac{\partial A_p}{\partial \tau} = E - (1 + i \Delta_p) A_p - A_s^2 + i D_p \nabla_{\perp}^2 A_p,$$

(4)

where $\nabla_{\perp}$ is the transverse Laplacian. $\Delta_{s,p}$ are the normalized detunings between the wave and the cavity mode, and the diffraction strengths are given by a weighted average over the layers:

$$D_s = \frac{c F}{2 \pi \omega} \left( \frac{l_{QC}}{n_{QC}} + \frac{l_{LHM}}{n_{LHM}(\omega/2)} \right), \quad D_p = \frac{c F}{4 \pi \omega} \left( \frac{l_{QC}}{n_{QC}} + \frac{l_{LHM}}{n_{LHM}(\omega)} \right),$$

(5)

where $F$ is the finesse of the cavity, $l_{QC}$ and $l_{LHM}$ are the lengths of the layers, and $n_{QC}$ and $n_{LHM}$ the indices of refraction. Eqs. (3)-(4) are obtained under the so-called mean-field approximations, which require (1) that reflections at the surfaces between the layers can be neglected; (2) that the dissipative Fresnel number is large; and (3) that the roundtrip length of the cavity is shorter than the diffraction, dispersion and nonlinearity space scales. Note that there is no phase matching in the LHM and, as a result, that there is no longer a fixed relationship between the two diffraction coefficients, unlike traditional DOPOs where $D_p = 2 D_s$. Finally, we want to mention that losses in the LHM can be accounted for in the mean-field model. In Ref. [32], it is shown that such losses naturally add to the cavity losses, and eventually result in a rescaling of the finesse of the cavity. By using a thin layer of LHM, the material losses will nevertheless be negligible to the cavity losses.

The homogeneous steady-state solutions of Eqs. (3)-(4) are (i) the nonlasing solution $A_s = 0$, $A_p = E/(1 + i \Delta_p)$ and (ii) the lasing solutions

$$A_s = \pm e^{i \theta} \sqrt{-1 + \Delta_s \Delta_p + \sqrt{|E|^2 - (\Delta_s + \Delta_p)^2}},$$

(6)

$$A_p = e^{i \theta} \sqrt{1 + \Delta_s^2}, \quad \theta = - \frac{1}{2} \arctan(\Delta_s + \Delta_p)/(|E|^2 - (\Delta_s + \Delta_p)),$$

The lasing solution emerges from the nonlasing solution in a pitchfork bifurcation at the lasing threshold $E_T = (1 + \Delta_s^2)^{1/2}(1 + \Delta_p^2)^{1/2}$. Note that the left-handed layer in the ring cavity does not modify the homogeneous steady states or the lasing threshold. However, as we will see in the following section, their stability is strongly affected by the presence of the left-handed layer.
3. Linear stability analysis and size scaling

A linear stability analysis of the homogeneous steady states (i) and (ii) with respect to finite wavelength perturbations of the form \(\exp(i\mathbf{k} \cdot \mathbf{r} - \lambda t)\), which are compatible with large Fresnel number systems, shows that the modulational (Turing) instability occurs when one of the eigenvalues of the linear operator vanishes. This happens when

\[
4|A_s|^2 \left[|A_s|^2 + Q_s(1 - Q_p) + Q_p\right] + Q_p(Q_s - |A_p|^2) = 0,
\]  

(7)

where \(Q_{s,p} = Q_{s,p}(k) = 1 + (\Delta_{s,p} + D_{s,p}k^2)^2\). Eq. (7) shows that the nonlasing solution (i) becomes unstable with respect to the modulational instability in the range \(E_M < E < E_T\), with \(E_M = (1 + \Delta_M^2)^{1/2}\). At the modulational instability point \(E = E_M\), the wavenumber is given by \(D_s k_M^2 = -\Delta_s\). The unstable wavelength at the modulational instability threshold is therefore

\[
\Lambda_M = 2\pi \sqrt{-D_s/\Delta_s}.
\]  

(8)

Negative diffraction of the signal field thus allows for dissipative structures in cavities with positive detuning \(\Delta_s > 0\). This is surprising since the pattern formation instability arises only for negative signal detuning in the absence of the LHM. To study analytically the stability of the lasing state (ii), let us assume that both signal and pump field are in perfect resonance, i.e., \(\Delta_s = \Delta_p\). Since we do not have to satisfy the phase matching condition, let us suppose that the diffraction coefficients of both fields are the same, i.e., \(D_s = D_p\). Under these assumptions, the critical wavelength is \(k_0^2 = -\Delta_s/D_s - \sqrt{(2|A_{SM}|^2 - 1 - \Delta_s^2)/(2(4|A_{SM}|^2 - 1))/D_s}\) and the threshold associated with the modulation instability, \(|A_{SM}|^2\), satisfies the following cubic equation

\[
64|A_{SM}|^6 + 48|A_{SM}|^4 - 16(1 + \Delta_s^2)|A_{SM}|^2 + (1 + \Delta_s^2)^2 = 0.
\]

This equation determines the critical field amplitude \(E_M\) at which the modulational instability takes place.

An important motivation for adding a LHM to nonlinear optical devices is the size scaling of the emerging dissipative structures made possible by the altered diffraction. It has been shown that dissipative structures can be scaled down beyond the diffraction limit by reducing the diffraction coefficient \([30]\), even though other effects can impose a new size limit \([33, 34]\). However, it is not straightforward that dissipative structures in an OPO would have similar scaling properties, since one cannot reduce the diffraction coefficients of the signal and the pump both at the same time. Indeed, most metamaterials are only left-handed in a narrow frequency band. Therefore, we have studied numerically how the dissipative structures in the DOPO scale in size when the signal diffraction coefficient \(D_s\) is decreased \((D_p\) is kept constant). We will treat structures in two regimes: stripes below and phase solitons above the lasing threshold.

\[\text{Fig. 2. Dissipative structures due to modulational instability below threshold. Signal diffraction strength is decreased from left to right: } D_s = -2.0, D_s = -1.0, D_s = -0.5, \text{ and } D_s = -0.3. \text{ Maxima are plain white and the mesh integration is } 128 \times 128.\]
For the stripes below the threshold, we have taken the parameters $E = 2.5$, $\Delta_s = \Delta_p = 2.0$, and $D_p = 1.5$. When $D_s$ is decreased (in magnitude) from $-2.0$ to $-0.3$, we indeed observe that the wavelength of the stripes becomes smaller [see Fig. 2(a)]. This figure is obtained from numerical simulations of the mean field model [Eqs. (3)-(4)]. The initial condition consists of a small amplitude noise added to the homogeneous steady state, and the boundary conditions are periodic in both transverse directions. We have calculated the period of the stripes by counting the number of stripes in the simulation area and we have plotted the result in Fig. 2(b). Further simulations have also shown that the pump diffraction $D_p$ has almost no influence on the size of the structures. This behavior can be understood from a perturbation analysis around the onset of modulational instability, which was presented in Ref. [32]. The pump field is homogeneous up to first order, implying a vanishing pump diffraction term in Eq. (4).

A similar analysis was performed for phase solitons above the lasing threshold, with parameters $E = 6.5$, $\Delta_s = -2.0$, and $D_p = 1.5$. Numerical simulations show labyrinthic structures [Fig. 3(a)]. By decreasing the signal diffraction strength, we observe the formation of domains in the form of phase solitons as shown in Fig. 3(a). Measuring the spatial size of the domains, we find again a size scaling proportional to the square root of the signal diffraction coefficient [Fig. 3(b)]. That the size scaling is valid without reducing the pump diffraction coefficient can here be explained by the phase indetermination of the two homogeneous lasing solutions; the phase solitons are made up of these two solutions. But since the pump field is actually the same for both lasing solutions, the pump diffraction term in Eq. (4) does not play a major role. This explains that the size can be scaled by adjusting only the signal diffraction coefficient.

4. Conclusion

We have studied a nonlinear resonator containing a LHM and a $\chi^{(2)}$ crystal. The LHM provides the ability to manage the diffraction of the signal field. We show that stable dissipative structures without diffraction matching ($D_p \neq 2D_s$) can exist for positive signal and pump detunings. In the absence of the LHM, these structures do not exist. The fact that our device can operate without diffraction matching also allows for the reduction of the wavelength of the emerging dissipative structures. Finally, we show that the size of the structures follows a square root law with respect to the signal diffraction coefficient.

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