RISK-BEARING, ENTREPRENEURSHIP AND THE THEORY OF MORAL HAZARD∗

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Abstract

The theory of moral hazard distinguishes two fundamental tradeoffs, one between risk sharing and incentives, and the other between surplus extraction and incentives. This paper isolates the former tradeoff and uses it to examine the “Knightian” theory of entrepreneurship, in which entrepreneurs provide insurance to workers by paying fixed wages, and profits are the residual from risk bearing. Existing models of entrepreneurship are extended by allowing for endogenous risk bearing not only via self-selection into occupations, but also through optimal insurance contracts. Moral hazard prevents full insurance; increases in the agent’s wealth then entail increases in risk borne. Thus, even under decreasing risk aversion, there are robust instances in which workers are wealthier than entrepreneurs. This empirically implausible result suggests that risk-based explanations for entrepreneurship are inadequate. Along with a related result showing that the more highly paid a worker is, the more he should be monitored, it can help to distinguish the empirical relevance of the two fundamental tradeoffs.

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1. Introduction

Entrepreneurship has long fascinated economists. The way we understand it affects our thinking about the processes generating growth and development, policies for influencing productivity and mitigating unemployment, even mechanisms underlying business cycles. One influential and intuitively appealing theory of the sources of and motivations for entrepreneurship can be traced back at least to Knight [1921], and has been articulated more recently by Kanbur [1979] and Kihlstrom and Laffont [1979]. In this theory, entrepreneurs — through the institution of the fixed wage contract — are viewed essentially as providers of insurance. Individuals choose between the safety of wages and the hazards of entrepreneurship according to their attitudes toward risk. More risk averse people (and with decreasing risk aversion, the poor) receive sure wages and work for the less risk averse (rich), who are the residual claimants. In the competitive version of this theory, everyone takes the wage as given, and it adjusts to make a marginal agent indifferent between the two occupations. An attractive feature of this theory (at least if we accept decreasing risk aversion) is that it easily explains one of the oldest stylized facts in economics, namely the tendency for entrepreneurs to be richer than workers.

Granting the basic presumption that being an entrepreneur is riskier than being a worker, this theory still suffers a major shortcoming: it assumes that the risks are exogenous, or more precisely that the choice of occupation is the only institutional arrangement available for risk sharing. Of course, there are alternatives, most prominent among them the market. Thus the first question we need to answer is why entrepreneurs should be bearing risk at all. One response is that the relevant risks are aggregate, and therefore cannot be insured away. Another is that even if risks are idiosyncratic, some information problem prevents full diversification. Either way, we are led to a second question: if we take proper account of the reason for the inability of the market to provide perfect insurance, does the Knightian theory still make plausible predictions?

In this paper, we focus on this second question, examining in particular how moral hazard on the part of entrepreneurs modifies the basic story (we shall briefly say something about alternative reasons for imperfect insurance at the end). It is straightforward to embed the risk sharing choices of entrepreneurs into a standard principal-agent framework. One of the drawbacks of that framework, though, is that it has led to few empirical predictions. Nonetheless, we are able
to show that for a broad class of utility functions, competitive equilibrium will entail that entrepreneurs are poorer than workers, rather than the other way around. Thus a plausible modification of the basic Knightian model leads to an implausible prediction. The fragility of this theory’s empirical predictions suggests that we probably should look elsewhere for explanations of the roles and causes of entrepreneurship.

The intuition behind the result is quite simple and depends on the existence of an apparently little-noticed property of the principal-agent model: when utility is separable in income and effort, the wealthier an agent is, the more risk he needs to bear in order to remain incentive compatible at a given effort level. The reason for this is that at low wealth levels, the income utility is very steep, so it only takes a small spread in the income “lotteries” generated by different effort levels to maintain a utility differential that will offset the disutility of effort. But as the income utility flattens with greater wealth, the spread in incomes must be increased in order to maintain the differential. Thus, wealthy agents need to bear more risk. Finally, for the class of utility functions we identify (namely those in which the marginal cost of providing utility is a convex transform of the utility function), this “increasing risk effect” swamps the decreasing risk aversion that goes with higher wealth, and we are led to the result.

The same intuition underlies a related result that we obtain for the standard formulation of the principal-agent model: agents with higher expected wages are monitored more than those with lower average wages. The evidence on this is perhaps less clear than that on the relative wealth of workers and entrepreneurs. Nevertheless, both results point to a larger issue, which is to sort out the relative empirical importance of the two fundamental trade-offs in the theory of moral hazard, namely that between risk sharing and incentives (which we focus on) and that between surplus extraction and incentives (the focus, for instance, of the efficiency wage literature). The textbooks seem to emphasize the risk sharing trade-off as the more fundamental. But based on the present results, it may be fair to conjecture that in real organizations, surplus extraction is what really matters.

2. A Model

Consider the following environment. There is a single good which is produced from labor according to the stochastic production function \( f(L, \theta) \), where \( L \) is the amount of labor hired (or worker effort) and \( \theta \) is a random variable indexing the state of the world, representing the risks that inhere in the production and sale of goods. Assume that raising \( \theta \) raises both the total and marginal products of labor, that is, \( f(L, \theta) \) and \( f_1(L, \theta) \) are increasing in \( \theta \) (for instance, \( \theta \) is a multiplicative noise). For the most part, we will assume that these risks are independent and identically distributed across firms. The production function
satisfies the standard properties: \( f(0, \theta) = 0, \ f_{11} < 0 < f_1, \lim_{L \to 0} f_1(L, \theta) = \infty, \lim_{L \to \infty} f_1(L, \theta) = 0. \) Each firm also requires the effort \( e \) of an entrepreneur. We can imagine that the entrepreneur expends this effort in coordinating production, marketing output, or perhaps monitoring workers. It enters the production technology only through the distribution of \( \theta \).

There is a continuum of agents, indexed by an interval, say \([0, 1]\). In order for the self selection underlying the Knightian story to have any relevance, agents must differ in some way. One way to do this is to focus on preferences, allowing, for instance, that all the variation be indexed by some utility parameter; this is the tack taken by Kihlstrom-Laffont (1979). Here we shall follow a special case of this approach, which is to assume that agents have identical preferences and vary instead in some other characteristic, namely initial wealth. Since standard assumptions, such as decreasing absolute risk aversion, then place restrictions on the relationship between risk attitudes and marginal incentives, this approach has the advantage of generating additional testable implications almost for free.

Thus, let all agents have identical preferences represented by the von Neumann-Morgenstern utility \( u(y) - e \), where \( y \) is realized income and \( e \) is the effort expended. The income utility \( u(\cdot) \) has the usual properties: \( u' > 0 > u'' \) with decreasing absolute risk aversion (i.e., \( u'''u' > (u'')^2 \)). In order to ensure that only risk sharing is at issue here, it is convenient, although by no means necessary, to assume that \( u(\cdot) \) is unbounded below.\(^1\) Agents vary in the amount of initial assets or wealth \( a \), the distribution of which is exogenous. An agent may choose one of two occupations in which to expend his effort: either he can become a worker, earning a sure wage \( w \) (so that his income is \( a + w \)), or he can be an entrepreneur, hiring \( L \) units of labor and earning the residual profit from production, which we denote \( y(\theta) \). Note that in general the amount of labor that an entrepreneur wishes to hire could depend not only on the wage, but also on his characteristics, which in this case is limited to his wealth \( a \).

Make the following assumptions on \( L, \theta, \) and \( e \): entrepreneurs may hire any nonnegative amount of labor, \( \theta \) is drawn from a finite set, and \( e \) is a binary variable taking values in \([0, 1]\) (the last assumption allows us to abstract from issues having to do with how effort levels might also change with parameters of the model). Denote by \( q(\theta) \) the probability of \( \theta \) when \( e = 1 \) and \( p(\theta) \) when \( e = 0 \); \( q(\cdot) \) stochastically dominates \( p(\cdot) \) and the two distributions have common support. We shall assume throughout that it is optimal for both workers and entrepreneurs to set their effort levels equal to 1.

Each agent takes the wage \( w \) as given and chooses the occupation which gives him the higher utility, taking into account any incentive compatibility constraints that may apply. Competitive equilibrium in this economy consists of a partition of the set of agents into a set \( W \) of workers and a set \( E \) of entrepreneurs, and a

\(^1\)The assumption ensures that no nonnegativity constraints on income bind at an optimum, which in turn purifies the model of any surplus extraction effect.
wage \( w \) such that the total demand for workers, given the wage, is equal to the total supply (see Kihlstrom-Laffont (1979) for a more formal definition).\(^2\)

We shall be concerned here only with characterizations of equilibrium, rather than its existence (Kihlstrom-Laffont deal with the latter issue). Nor shall we be worried about financing (i.e., we assume entrepreneurs always have enough income ex-post to pay their workers; with utility unbounded below, this will never be a problem), so that we can focus purely on the role of risk aversion in the occupational choice.

3. No Insurance for Entrepreneurs

This is the case considered in the literature. It is convenient to represent the indirect utility of an agent as function of his wealth and occupation; let \( V_E(a) \) be the utility achievable by an agent with wealth \( a \) if he becomes an entrepreneur, and \( V_W(a) \) be the utility he achieves as a worker. Since by assumption, being a worker is safe, he obtains the income \( w + a \), and \( V_W(a) = u(w + a) - 1 \).

Denote by \( y(\theta) \) the income of an entrepreneur in state \( \theta \). In the absence of any insurance for entrepreneurs, we have

\[
y(\theta) = f(L, \theta) - wL + a \quad \text{and} \quad V_E(a) = \max_L E|_{e=1}\{u(f(L, \theta) - wL + a)\} - 1 \quad \text{(we assume that it is always optimal for the entrepreneur to set } e = 1)\]

where the expectation is taken with respect to the distribution of \( \theta \), conditional on \( \theta \).

When contemplating becoming an entrepreneur, an agent will choose \( L \) to maximize his expected utility, taking as given the wage \( w \). Since a larger firm will generate a larger spread among the possible output realizations, we might expect that entrepreneurs will attempt to self-insure by varying the sizes of their firms. Indeed, it is easy to show that the wealthier — and therefore the less risk averse — an entrepreneur, the larger will be his firm: the argument is a standard one from the theory of portfolio choice, and depends on the assumptions of decreasing risk aversion and on the properties of the production function stated above.\(^3\) Since the expected profit \( E|_{e=1}\{f(L, \theta) - wL\} \) is increasing when \( L \) is

\(^2\)For completeness, we offer a slightly less general definition which is valid if the distribution of initial wealth is represented by an atomless measure \( \eta(a) \). Suppose an entrepreneur with wealth \( a \) demands \( L(w; a) \). An equilibrium is then a partition of the wealth interval into sets \( A_E \) and \( A_W \) such that

\[
\int_{A_W} d\eta(a) = \int_{A_E} L(w; a)d\eta(a).
\]

\(^3\)To see this, differentiate the first order condition for \( L \)

\[
E|_{e=1}\{u'(f(L, \theta) - wL + a)(f_1(L, \theta) - w)\} = 0
\]

and observe that \( \frac{dL}{da} \) has the same sign as \( E|_{e=1}\{u''(f(L, \theta) - wL + a)(f_1(L, \theta) - w)\} \). Now \( f_1 - w \) is monotonic and must be negative for low values of \( \theta \) and positive for high values of \( \theta \) if the
below its first-best level, it follows that entrepreneurs’ profits are increasing in wealth. This model therefore captures the old idea that profits are a return to risk-bearing. It also provides an account for the observation that firms (or plants) in the same industry vary in size.

In equilibrium there is a wealth level \( \bar{a} \) at which an agent is indifferent between the two occupations (since \( V_W(a) \) and \( V_E(a) \) are continuous functions of \( a \), if there were no such \( \bar{a} \), everyone would prefer one of the occupations, which cannot be an equilibrium). We now argue that this point is unique, and that all agents who become workers are below \( \bar{a} \), while all entrepreneurs are above it. To see this, note that at \( \bar{a} \), \( u(\bar{a} + w) = E|\epsilon=1\{u(f(L, \theta) - wL + \bar{a})\} \), that is, the worker’s income \( w \) is the certainty equivalent of the entrepreneur’s income \( f(L, \theta) - wL \) for this marginal agent. Consider an agent with wealth \( a_0 \) slightly greater than \( \bar{a} \), and suppose she contemplated being an entrepreneur using the same amount of labor as the agent with \( \bar{a} \): since the agent with \( a_0 \) is less risk averse than the agent with \( \bar{a} \), she strictly prefers the lottery (i.e. the entrepreneur’s income) to the worker’s safe income; since as an entrepreneur agent \( a' \) would typically choose some level of \( L \) different from that chosen by \( \bar{a} \), she prefers entrepreneurship all the more strongly. This argument implies that \( V_E(a) \) cuts \( V_W(a) \) from below wherever they are equal (in other words, \( V'_E(\bar{a}) > V'_W(\bar{a}) \) ), and so \( \bar{a} \) is unique.

To summarize, we have the following results, which are virtually the same as those in Kihlstrom-Laffont (1979):

**Proposition 1.** When entrepreneurs cannot insure, in competitive equilibrium the first-order condition is to be satisfied. Let \( \hat{\theta} \) be a value for which \( f_1 - w < 0 \) for \( \theta < \hat{\theta} \) and \( f_1 - w > 0 \) for \( \theta > \hat{\theta} \). Then by decreasing risk aversion,

\[
-\frac{u''(y(\theta))}{u'(y(\theta))} > -\frac{u''(y(\hat{\theta}))}{u'(y(\hat{\theta}))}, \quad \theta < \hat{\theta};
\]

upon multiplying both sides by \( -q(\theta)u'(y(\theta))(f_1(L, \theta) - w) \) and adding up to \( \hat{\theta} \), one obtains

\[
\sum_{\theta < \hat{\theta}} q(\theta)u''(f_1 - w) > \frac{u''(y(\hat{\theta}))}{u'(y(\hat{\theta}))} \sum_{\theta < \hat{\theta}} q(\theta)u'(f_1 - w).
\]

Similarly, one can show that

\[
\sum_{\theta > \hat{\theta}} q(\theta)u''(f_1 - w) > \frac{u''(y(\hat{\theta}))}{u'(y(\hat{\theta}))} \sum_{\theta > \hat{\theta}} q(\theta)u'(f_1 - w).
\]

Taken together, these imply

\[
Eu''(f_1 - w) > \frac{u''(y(\hat{\theta}))}{u'(y(\hat{\theta}))}Eu'(f_1 - w) = 0,
\]

where the equality follows from the first-order condition.
(1) Firm size — as measured by the amount of labor demanded — and expected profit increase with the entrepreneur’s initial wealth;

(2) There exists a wealth level $\bar{a}$ such that all workers have wealth at most $\bar{a}$ and all entrepreneurs have wealth at least $\bar{a}$.

4. The Second-Best Economy with Moral Hazard

Proposition 1 makes the empirically plausible predictions that wealthy agents will tend to become entrepreneurs and poor ones workers, and accounts for variations in firm size according to the risk preferences of their owners. But observe that even though entrepreneurs are less risk averse than workers, they are still risk averse, and would be better off if they could share risks, say through a stock market. The model of the previous section has arbitrarily excluded them from engaging in these contracts. In this section, we consider one possibility for an improved risk sharing arrangement, one which nonetheless leaves some risks for entrepreneurs to bear.

There are, of course, several reasons why entrepreneurs might bear some (idiosyncratic) risk. Predominant among them is moral hazard — the entrepreneur’s activities are not verifiable to the outside world. We shall now consider the optimal risk-sharing arrangement when the entrepreneur’s effort is not observable (workers’ effort and the level of $L$ is assumed to be verifiable, however).

Assuming free entry into the insurance market and that each entrepreneur’s contract satisfies incentive compatibility, the optimal risk sharing scheme can be written as a function $y(\theta)$ which satisfies (again assuming that $e = 1$ is optimal; if it were not, then full insurance would always be possible, making entrepreneurship as safe as working, and the Knightian theory would be irrelevant)

$$\max_{y(\theta), L} \sum q(\theta)u(y(\theta)) - 1$$

s.t. $\sum q(\theta)y(\theta) \leq \sum q(\theta)f(L, \theta) - wL + a$ \hspace{1cm} (4.2)

$$\sum q(\theta)u(y(\theta)) - 1 \geq \sum p(\theta)u(y(\theta))$$

(4.3)

Let $\hat{V}_E(a)$ denote the value of this problem and $(\hat{y}(\theta), \hat{L})$ its solution. At an optimum, both constraints hold as equalities: that (4.3) binds is a standard result in the principal-agent literature (see e.g., Hart-Holmström, 1987). As for (4.2), if it did not bind, it would be possible to raise the $y(\theta)$ in such a way as to increase utility by the same amount in each state. This in turn raises the payoff without affecting (4.3), a contradiction. Let us now examine properties of the solution in somewhat greater detail.

The first observation to make is that, regardless of initial wealth (or risk attitude), all entrepreneurs will choose the same size of firm $\hat{L}$, namely that which
maximizes $\sum q(\theta)f(L, \theta) - wL$ (following the argument in the previous paragraph, increasing the expected profit, thereby relaxing (4.2), allows the entrepreneur to raise his payoff). This scale of operation is in fact the first-best level which we denote $L^*$. Immediately, then, we have a departure from the closed-market formulation considered in the previous section. Expected profits are also independent of the entrepreneur’s wealth.

We shall also use the following

**Lemma 1.** Let $\hat{V}_E(a)$ be the value function for problem (4.1). Then

$$\frac{1}{\hat{V}_E'(a)} = \sum q(\theta) \frac{1}{u'(\hat{y}(\theta))}$$

**Proof.** Let $\alpha$ and $\beta$ denote the multipliers on the constraints (4.2) and (4.3) respectively (as we pointed out above, they both bind at an optimum). The first-order condition with respect to $y(\theta)$ can be written

$$\frac{q(\theta)}{\alpha} = \frac{q(\theta)}{u'(\hat{y}(\theta))} - \frac{\beta}{\alpha} (q(\theta) - p(\theta))$$

adding over $\theta$ gives

$$\frac{1}{\alpha} = \sum q(\theta) \frac{1}{u'(\hat{y}(\theta))}.$$  

Noting from the envelope theorem that $\hat{V}_E'(a) = \alpha$ completes the argument. ■

This result holds even if there are many effort levels, provided utility is additively separable in income and effort: the same proof applies with only minor modification. It simply states that at an optimal contract, the marginal cost of providing the expected utility $V_E(a)$ to the entrepreneur (the left-hand side) is equal to the expected marginal cost of providing him with utility $u(\hat{y}(\theta))$ in each state (right-hand side; recall that the cost in terms of output of raising utility a small amount from $u(y(\theta))$ is $\frac{1}{u'(\hat{y}(\theta))}$). A related result appears in Rogerson (1985).

4.1. Occupational Self-Selection in the Second-Best Economy

We have already noted one departure of the second-best scheme from the closed-market scheme, namely the absence of a scale effect. We now show that there is likely to be a second, dramatic departure in the way agents self-select into occupations.

Make one additional restriction on the income utility:

$$\frac{1}{u'(x)} = h(u(x))$$

for some strictly convex $h$. (4.4)
The role of this assumption will be discussed below. Suffice for now to say that it is satisfied by a broad class of utility functions; of those in the constant-relative-risk aversion (CRRA) class \( \frac{y^{1-\sigma}}{1-\sigma} \), for instance, all utility functions with \( \sigma > 1/2 \) satisfy (4.4) (indeed, in the CRRA case, the condition is automatically entailed in the requirement that \( u(\cdot) \) is unbounded below, for then \( \sigma \geq 1 \)).

Once again the strategy is to compare the slopes of the value functions for the two occupations at a point of indifference between them. Just as in the case with no insurance, equilibrium entails that such a point exists; call it \( \hat{a} \). Since for an agent with \( \hat{a} \), being a worker and expending a unit of effort generates the same utility as being a (risk-bearing) entrepreneur and expending a unit of effort, we have

\[
 u(w + \hat{a}) = \sum q(\theta) u(\hat{y}(\theta)).
\]

In other words, for the marginal agent, the worker’s income is the certainty equivalent of the entrepreneur’s. The well-known theorem of Arrow and Pratt tells us that anyone who is less risk averse than a consumer with utility function \( u(\cdot) \) will strictly prefer the entrepreneur’s income lottery. In other words, for any strictly convex increasing \( h(\cdot) \),

\[
 h(u(w + \hat{a})) < \sum q(\theta) h(u(\hat{y}(\theta))).
\]

But since by (4.4) we have \( 1/u'(\cdot) = h(u(\cdot)) \) for some such \( h(\cdot) \), we have:

\[
 \frac{1}{V_W'(\hat{a})} = \frac{1}{u'(w + \hat{a})} < \sum q(\theta) \frac{1}{u'(\hat{y}(\theta))} = \frac{1}{V_E'(\hat{a})}. \tag{4.5}
\]

(The first equation follows from a simple computation, and the second is from Lemma 1.) Thus we have

\[
 V_W'(\hat{a}) > V_E'(\hat{a}),
\]

and we conclude that in the equilibrium of the second-best economy entrepreneurs are poorer than workers!

The principal properties of equilibrium in the second-best economy are summarized in

**Proposition 2.** In a second-best equilibrium:

1. Both firm size and expected profit are equal to their first-best levels and are independent of the entrepreneur’s initial wealth;
2. There exists a wealth level \( \hat{a} \) such that all entrepreneurs have wealth at most \( \hat{a} \) and all workers have wealth at least \( \hat{a} \).

What this proposition shows is that the Knightian theory, when pushed to a reasonable standard of theoretical consistency, predicts that quite commonly we should find that the poor hiring the rich. Since this is empirically implausible,
we might call into question theories which view the primary determinant of who becomes an entrepreneur to be risk attitudes and the primary function of the entrepreneur as an insurer of workers. In particular, it seems that the evidence suggests that the risk-sharing-versus-incentives-trade-off is not the primary one operating in the occupational choice between entrepreneur and worker.

4.2. Analysis of the Main Result

Why the dramatic difference in predictions about the occupational choice? Some intuition for the result is contained in the following proposition, which states that as an entrepreneur’s wealth increases, he needs to bear more risk in order to remain incentive compatible.4

Proposition 3. Let \( v(a) \) be the variance of the least-risk contract satisfying the binding forms of (4.2) and (4.3), i.e.

\[
v(a) = \min_{y(\theta)} \sum q(\theta)[y(\theta) - \sum q(\theta)y(\theta)]^2
\]

s.t. \( \sum q(\theta)y(\theta) = \sum q(\theta)f(L^*, \theta) - wL^* + a \)

\[
\sum q(\theta)u(y(\theta)) - 1 = \sum p(\theta)u(y(\theta)).
\]

Then \( v'(a) > 0 \).

Proof. Let \( \gamma \) be the multiplier on the first constraint and \( \delta \) be the multiplier on the second. Then the envelope theorem tells us that \( v'(a) = -\gamma \). We need only show that \( \gamma < 0 \). The first-order condition for this problem can be written

\[
2 \frac{q(\theta)(y(\theta) - X)}{u'(y(\theta))} = -\frac{\gamma q(\theta)}{u'(y(\theta))} + \delta(q(\theta) - p(\theta)),
\]

where \( X \equiv \sum q(\theta)f(L^*, \theta) - wL^* + a \). Adding over \( \theta \) yields

\[
2 \sum \frac{q(\theta)(y(\theta) - X)}{u'(y(\theta))} = -\gamma \sum \frac{q(\theta)}{u'(y(\theta))}.
\]

Thus \( \gamma \) is negative if and only if \( \sum \frac{q(\theta)(y(\theta) - X)}{u'(y(\theta))} > 0 \). Now, for all \( \theta \) such that \( y(\theta) > X \), we have, by concavity of \( u(\cdot) \), \( \frac{q(\theta)(y(\theta) - X)}{u'(y(\theta))} > \frac{q(\theta)(y(\theta) - X)}{u'(X)} \); similarly, for all \( \theta \) such that \( y(\theta) < X \), we also have \( \frac{q(\theta)(y(\theta) - X)}{u'(y(\theta))} > \frac{q(\theta)(y(\theta) - X)}{u'(X)} \). Therefore,

\[
\sum \frac{q(\theta)(y(\theta) - X)}{u'(y(\theta))} > \sum \frac{q(\theta)(y(\theta) - X)}{u'(X)} = 0.
\]

\( 4 \)Subject, of course, to the well-worn caveat that increases in variance do not necessarily entail increases in risk in the sense of second-order stochastic dominance.
Observe that the only properties (besides differentiability) of the utility used in the proof are that \( u(\cdot) \) is increasing and concave — in particular, no third-derivative conditions are required.\(^5\) The intuition for the result is very simple: at low wealth levels the income utility is very steep, so a small spread among the \( y(\theta) \), when weighted by the different distributions \( q(\cdot) \) and \( p(\cdot) \), will be enough to maintain a utility differential that will offset the disutility of effort. As the income utility flattens, the spread in incomes must be increased in order to maintain the differential. Thus, wealthy agents need to bear more risk.

Note that the result is a statement about the entrepreneur’s constraints rather than about the optimum: it does not quite say that a wealthy entrepreneur will bear greater risks than a poor one (since the poor entrepreneur might choose a high variance contract even though he does not have to), although there are cases where this will be true (for instance, if there are only two states).

It is this increasing risk effect, which is not present in the closed market case, where the required risk for a given size of firm (as opposed to the chosen risk which changes because the firm size does) is constant, that leads to the result in Proposition 2. If the risk increases fast enough, compared to the rate at which risk aversion decreases, then the first effect dominates.

Now we can understand the role of condition (4.4). It is essentially a requirement that risk aversion not decline too quickly and therefore guarantees that this trade-off goes at the right speed: letting \( \rho \) denote absolute risk aversion, an equivalent way to write the condition is

\[
\frac{\mu}{\rho} \frac{d\rho}{dy} \leq \frac{y}{(1/u)^2} \frac{d(1/u)^2}{dy}.
\]

Meanwhile, recall that the nonnegative profit condition permits the entrepreneur to choose the first-best scale for her firm. In a first-best world, of course, the entrepreneur would have the entire half-space from which to choose; here she is confined to a (full-dimensional) subset of that set. Moral hazard doesn’t really cause markets to close so much as it leaves them “half-open.”\(^7\)

\(^5\)The result is also valid for multiple effort levels, at least if the “first-order approach” is valid (almost exactly the same proof applies), in which case it is properly interpreted to say that the amount of variance needed to implement a given effort level is increasing in the agent’s wealth.

\(^6\)In fact, this condition has appeared in the literature before, albeit in a different context. It is among the suite of conditions provided by Jewitt (1988) to justify use of the first-order approach. Since the first order approach has generally been assumed to be valid for most of the informativeness orderings that have been proposed for the principal-agent model, the conjecture about the complementarity of monitoring and outside options made in the next subsection, if correct, would then be true without any special assumptions.

\(^7\)Also note that in contrast to the first-best and closed-market cases, the extent of the agent’s trading opportunities (that is, how much of the half-space is available to him), depend on his (verifiable) characteristics — in this case his wealth. When there are more than two outcomes (say \( S \) of them), the difference between the closed-market and second-best case is dimensional: with no insurance market, the entrepreneur, by varying \( L \), can choose from a one-dimensional set of points (ignoring free disposal). But when he can insure — even if limited by the incentive constraints of moral hazard — his choices form an \( S - 1 \) dimensional set.
4.3. Monitoring and Task Assignment

Lemma 1 has other implications. Consider for instance the standard Principal-Agent setting in which the agent is now interpreted as an employee and the principal is a firm. We are interested in how the firm will choose the level of monitoring it applies to the worker.

To model this, assume that there is a family of monitoring technologies indexed by \( m \). Continue to assume that a worker can choose from among two effort levels with probability distributions \( q \) and \( p \). We assume that higher \( m \) implies a higher level of monitoring, i.e., that the higher is \( m \), the more efficient is the information system \( \{q(\theta, m), p(\theta, m)\} \).\(^8\) For a fixed level of \( m \) and a worker whose outside option yields utility \( u \), the firm minimizes the cost of a worker \( C(u, m) \), i.e.

\[
C(u, m) \equiv \min_{y(\theta)} \sum q(\theta, m)y(\theta)
\]

\[
\text{s.t. } \sum q(\theta, m)u(y(\theta)) - 1 \geq u
\]

\[
\sum q(\theta, m)u(y(\theta)) - 1 = \sum p(\theta, m)u(y(\theta)).
\]

First observe that Lemma 1 applies to this problem and can be written \( \frac{\partial C(u, m)}{\partial u} = \sum \frac{q(\theta, m)}{u(y(\theta))} \). More efficient information implies that \( C(u, m) \) is decreasing in \( m \). Moreover, it is clear that \( C(u, m) \) is increasing in \( u \): workers with higher outside options receive higher average wages.

Now suppose that the firm has a series of equally productive tasks and that it is assigning workers with different outside options to them. The tasks differ in the amount of information that is available about the effort expended on them (for instance, one task might be a production task which is relatively easy to observe while the other might be administrative and therefore more difficult to observe); workers are equally capable at the tasks (more generally, the outside option \( u \) is uncorrelated with the worker’s cost or the expected revenue generated by the worker at each effort level) and the question is which workers should be assigned to which task. Thus we assume that the tasks are ordered by \( m \) as above. It is a well-known consequence of the theory of assortative matching that if the firm is trying to minimize its total cost, the worker with the higher \( u \) will be assigned to the task with higher \( m \) if \( C(u, m) \) is submodular in \( (u, m) \).

Alternatively, suppose that the firm can choose the level of costly monitoring to be expended on a worker: more monitoring means a higher level of \( m \). Assume

\(^8\)Several rankings of information systems have been studied in the incentive literature (see, e.g. Holmstrom, 1979; Kim, 1995; Jewitt, 1999). For present purposes, it does not matter which of these we use.
that monitoring costs are increasing and convex in \( m \). Then if \( C(u, m) \) is submodular, the firm will choose to monitor workers more intensively if they have a higher level of \( u \).

Now suppose that \( u(y) = \ln y \). Then \( 1/u' = y \) and it follows that \( \sum q(\theta, m) y(\theta) = C(u, m) = \frac{\partial C(u, m)}{\partial u} \). Thus if \( m > \hat{m} \), we have \( \frac{\partial C(u, m)}{\partial u} < \frac{\partial C(u, \hat{m})}{\partial u} \)

which yields

**Proposition 4.** Suppose that workers have logarithmic utility for income. Then workers with higher average wages are assigned to more easily observed tasks and are monitored more intensively.

Although we have shown this to be true in the logarithmic case (i.e. logarithmic utility is sufficient for submodularity of \( C(u, m) \)), it certainly holds for a broader class of utility functions, and we conjecture that the class satisfying (4.4) will give the same result.

While we are somewhat skeptical of the empirical plausibility of either implication of this proposition, we are not aware of systematic evidence on this question. The main point is that the result does offer a possible avenue for research on the empirical significance of the risk-sharing/incentives tradeoff.

5. Discussion

We have examined one prominent interpretation of the Knightian idea that entrepreneurship is a form of risk sharing and profits a return to risk-bearing and have shown that, when properly specified, it can easily lead to implausible predictions. This suggests that alternative approaches are more likely to be useful for understanding entrepreneurship.\(^9\)

Besides illustrating the general principle that closing a market is not the same as leaving it “half open,” the analysis here underscores the importance of “matching” or selection effects in the study of contracts and organization (e.g. Ackerberg-Botticini, 2002). Notice that a straightforward interpretation of Proposition 3 would say that as agents grow wealthier, the amount of risk they bear increases (subject to the caveat noted above). But Proposition 2, which takes explicit account of the character of agents’ outside opportunities (in this case the necessity that at least some of them take the wage contract, which in turn implies the existence of the “marginal” wealth levels \( \bar{a} \) and \( \hat{a} \)) predicts almost

\(^9\)A couple come to mind. One is based on credit market imperfections (which in turn depend on a property of the agent’s payoff quite different from risk aversion, namely whether it is bounded below), which can easily explain both why the wealthy bear more risk and why they tend to be the entrepreneurs (Banerjee-Newman, 1993).

Another looks at “ability” (Lucas, 1978; Lazear, 2002), which is potentially fruitful to the extent that it can be measured and its effects disentangled from skills that might be acquired in the market (the credit market imperfection approach would apply to the latter).
the opposite: while increasing risk occurs over a range of low wealth levels, the overall effect is of an “inverted U,” with the rich agents bearing no risk at all.

We turn now to a discussion of robustness of the main result.

5.1. Multiple Effort Levels

In the standard separable-utility specification of the principal-agent model, leisure (i.e. reduction of effort) is a normal good. If one assumes that effort is a continuous variable and that its cost is strictly convex, then as the agent’s wealth increases, he will lower his effort, thereby reducing its marginal cost; this in turn weakens the increasing-risk effect which was essential to the poor-hire-the-rich result in Proposition 2. One then wonders whether the result is simply a consequence of the two-effort level formulation we have used.

There are two approaches to this question. First, we could try to characterize equilibria of a multiple-effort-level version of the model explicitly. In general, this appears to be a difficult exercise. We do have an example in which the probabilities and effort cost are linear in \( e \in [0, 1] \) where entrepreneurs behave as in the two effort case, choosing \( e = 1 \), while workers choose effort in the interior of the interval. The existence of a marginal agent who is indifferent between the two occupations now implies that his income as a worker is strictly lower than his expected income as an entrepreneur, and under the assumption that \( 1/u' \) is convex, the chain of inequalities (4.5) remains valid (recall that the truth of Lemma 1 doesn’t depend on the number of effort levels). Thus, in this case at least, the configuration of occupational choices is as stated in Proposition 2. Moreover, for strictly convex effort costs which are “close” to linear, the equilibrium configuration remains the same. See the Appendix.

An alternative response also exploits the fact that Lemma 1 is valid for any number of effort levels. Suppose that \( u(y) = \ln y \) (and so satisfies condition (4.4)); effort cost is \( c(e) \), where \( c' \geq 0, c'' > 0, c(0) = c'(0) = 0 \); a worker who exerts \( e_W \geq 0 \) earns \( e_W w \); and that if an entrepreneur expends \( e_E \), the probability of state \( \theta \) is \( h(e_E, \theta) \), where the distributions \( h(\cdot, \cdot) \) have common support and increases in \( e_E \) result in increases in the distribution of \( \theta \) in the sense of first-order stochastic dominance. Denote the expected profits of the marginal agent when he is an entrepreneur by \( E_{eE} \pi \). Again, since for \( u(y) = \ln y, 1/u'(y) = y, \) Lemma 1 implies that \( V_{eE}'(\hat{\alpha}) > V_{W}'(\hat{\alpha}) \) if and only if \( e_W w + \hat{\alpha} > E_{eE} \pi + \hat{\alpha} \). Now, effort is nonincreasing in wealth for both workers and entrepreneurs (this is easy to check from the first-order condition for workers, but is somewhat more complicated for entrepreneurs — see the Appendix). This implies that the wealthier a worker is, the less he earns; similarly, the wealthier is an entrepreneur, the lower are his profits on average (given the specified family of distributions, effort and labor are complements, so lower effort corresponds to a smaller and less profitable firm).

Suppose then we have the realistic case in which the rich hire the poor. The argument we just made gives us the following
Proposition 5. Suppose income utility is logarithmic and there is a wealth level $a^*$ such that all entrepreneurs have wealth at least $a^*$ and all workers have wealth at most $a^*$. Then every worker has higher (expected) earnings than every entrepreneur.

This result suggests that allowing for multiple effort levels in the Knightian model need not reduce the likelihood of empirically implausible predictions.  

5.2. Other Interpretations of the Knightian Theory

The basic predictions of the Knightian theory are not robust to a moral hazard interpretation. What happens in the presence of other causes of imperfect insurance?

One alternative interpretation of the risk-bearing story is that entrepreneurs bear the aggregate risks, rather than idiosyncratic ones. Then if one assumes that there is no possibility for entrepreneurs to insure against these risks, while workers are perfectly insured, one can easily show that the poor (and highly risk averse) agents become workers while the wealthy (and less risk averse) become entrepreneurs.

But once again, this fails to correspond to optimal risk sharing. For instance, with CRRA utility, it is not hard to show that if there is only aggregate risk, everyone — worker and entrepreneur, rich and poor — will bear a risk that is proportional to his initial endowment. In this case, the problem is not so much that the rich end up bearing smaller risks than the poor (they don’t), it is that — from the risk-bearing point of view, at least — there is no meaningful distinction between the two occupations.

Adverse selection is another reason for incomplete insurance. Perhaps entrepreneurs know more about the likelihood of success of their ideas than do outsiders. Specifically, suppose for simplicity that projects are of fixed size, and have two outcomes. Projects come in two qualities, measured by the probability of success; the quality of a project is private information. In this case, one can use a screening model of the Rothschild-Stiglitz (1976) variety to show that in equilibrium, regardless of wealth level (or risk attitude) agents with high quality projects become entrepreneurs, while agents with low quality projects are screened off and become workers. This may be a socially efficient and perhaps even plausible outcome, but it has nothing to do with risk attitudes (the same result could come from a model in which everyone is risk neutral).

Finally, we could depart from the rational expectations framework and suppose that people have different beliefs about their success which they don’t completely revise in the light of market-transmitted information. In this case, though,

\[\text{If there are multiple crossings of the value functions, then things are not so simple. But once again, we are forced to accept the predictions that either the poorest members of the economy are entrepreneurs who hire people wealthier than they, or the agents with the lowest initial wealth earn the highest incomes.}\]
it is easy to construct “Edgeworth Box” examples where markets operate perfectly but in which a highly optimistic but risk averse trader bears all the risk while a more guarded but risk neutral one bears none, opposite the predictions of the Knightian model.

5.3. Implications for the Theory of Moral Hazard

Finally, we wish to suggest that the results in this paper have broader implications for the theory of moral hazard, at least as it expressed in the principal-agent model. A common view (emphasized in textbooks) seems to be that “the” fundamental trade-off is the one between the provision of incentives and the sharing of risk. But principal-agent theory has also revealed a trade-off between incentives and surplus extraction: if, for instance, there is a lower bound on the utility that the agent can receive (contrary to what we have assumed in order to isolate the risk sharing aspects), it may be necessary to pay him a rent in order to maintain incentives (the efficiency wage literature is perhaps the most prominent application of this idea, although the same kind of mechanism is fundamental to many models of imperfect credit markets). The distinction between these trade-offs is crucial for understanding the role of organizations, and for evaluating their economic performance. Which of them is more relevant to understanding our world is of course an empirical question, and it is therefore important to derive empirical implications of these trade-offs. Though the evidence is hardly conclusive, the results in this paper suggest that in fact the risk sharing trade-off may be less relevant: in the real world of organizations, moral hazard is more about surplus extraction than it is about risk sharing.

6. Appendix

Here we elaborate on the arguments in Section 5.1.

A Linear Example. Suppose that effort is chosen from the unit interval and that its disutility is linear. Denote the income of a worker with wealth \( a \) by \( y_W(\hat{a}) \) and the effort level he chooses by \( e_W(\hat{a}) \). For the marginal agent with wealth \( \hat{a} \),

\[
u(y_W(\hat{a})) - e_W(\hat{a}) = E v(y(\theta)) - e_E(\hat{a}),\]

where \( y(\theta) \) is the entrepreneur’s income function and \( e_E \) his effort. Thus if \( e_E(\hat{a}) \geq e_W(\hat{a}) \), we deduce that \( u(y_W(\hat{a})) \leq E u(y(\theta)) \) and then use (4.5) to conclude that the entrepreneurs are poorer than the workers.

Suppose that when the entrepreneur chooses \( e \), the probability of state \( \theta \) is \( eq(\theta) + (1 - e)p(\theta) \), where \( q(\cdot) \) stochastically dominates \( p(\cdot) \) and the two distributions have common support (we shall suppress notating the dependence on \( \theta \) in what follows). The entrepreneur’s problem is
\[
\max_{y(\theta), e} e \sum (q - p)u(y) + \sum pu(y) - e
\]

s.t. \( e \sum (q - p)y + \sum py = \max_L e \sum (q - p)f(L) + \sum pf(L) - wL + a \) (6.1)

\[\begin{align*}
e \sum (q-p)u(y) + \sum pu(y) - e & \geq e' \sum (q-p)u(y) + \sum pu(y) - e', \forall e' \in [0, 1]. \tag{6.2}
\end{align*}\]

We now show that any nontrivial solution to this problem entails \( e_E = 1 \) for all \( a \), and therefore \( e_E \geq e_W \). Observe that (6.2) can be written

\[
e \sum (q-p)u(y) - 1 \geq e'[\sum (q-p)u(y) - 1];
\]

thus \( \sum (q-p)u(y) \geq 1 \) (else \( e = 0 \)). If \( \sum (q-p)u(y) > 1 \), then \( e = 1 \) is the only one satisfying (6.2), and we are done. If instead \( e \in (0, 1) \), then \( \sum (q-p)u(y) = 1 \). Raising \( e \) until it is equal to 1 therefore leaves (6.2) unchanged. If it relaxes (6.1), then utility in each state can be raised without affecting (6.2), contradicting the assumption that \( e < 1 \) is optimal. Increasing \( e \) relaxes (6.1) provided \( \sum (q-p)(f(L(e)) - y) > 0 \) (here \( L(e) \) denotes the profit maximizing choice of \( L \) given \( e \)); we claim this condition must hold. Suppose not. From (6.1), \( \sum py \leq \sum pf(L(e)) - wL(e) + a \). Then

\[
\sum pu(y) \leq u(\sum py) \leq u(\sum pf(L(e)) - wL(e) + a) \leq u(\sum pf(L(0)) - wL(0) + a);
\]

where \( (a) \) is by risk aversion and \( (c) \) is from the fact that \( L(0) \) maximizes \( \sum pf(L) - wL. \) Since \( \sum (q-p)u(y) = 1 \), the entrepreneur’s payoff is \( \sum pu(y) \); on the other hand, receiving \( \sum pf(L(0)) - wL(0) + a \) in every state is feasible, and we are thus led to a contradiction.

This result shows that at every equilibrium wage of the economy with linear utility, the occupational configuration is as in Proposition 2. Now consider perturbations of this economy of the following sort. Replace the cost function with one that is of the form \( c_\lambda(e) = (1 - \lambda)e + \lambda C(e) \), where \( C(e) \) is some smooth, strictly convex, strictly increasing function. Since value functions are continuous in the parameters, the relative slopes of \( V_E(a) \) and \( V_W(a) \), thought of as functions of \( \lambda \) and \( w \), will not change for values of \( \lambda \) in a neighborhood of zero. The direct effect of changing \( \lambda \) slightly will be small, and the indirect effect, through the change in \( w \), will also be small: for some interval \((0, \bar{\lambda}) \), every equilibrium wage of an economy in which the cost function is \( c_\lambda(e), \lambda \in (0, \bar{\lambda}) \), must be close to one in the original economy. Thus for this family of strictly convex costs, at least, the main result in Proposition 2 is unchanged.
Entrepreneurial Effort Decreases with Wealth. The entrepreneur solves

\[ \max_e G(e, a), \]

where

\[ G(e, a) \equiv \max_{\{y(\theta)\}} \sum h(e, \theta)u(y(\theta)) - c(e) \]

subject to

\[ \sum h(e, \theta)y(\theta) = \max_L \sum h(e, \theta)f(L, \theta) - wL + a \equiv \pi(e) + a \]

\[ \sum h(e, \theta)u(y(\theta)) - c(e) \geq \sum h(e', \theta)u(y(\theta)) - c(e') \forall e'. \]

By Topkis’s and Milgrom-Shannon’s results on supermodular optimization (Milgrom-Shannon, 1994), it is enough to show that \( \partial^2 G/\partial e \partial a < 0 \) in order to guarantee that \( e_E \) is nonincreasing in \( a \). From the envelope theorem and Lemma 1, \( \partial G/\partial a = 1/\sum h(e, \theta)/u(y(\theta)) \). Thus \( \partial^2 G/\partial e \partial a < 0 \) if and only if \( \sum h(e, \theta)/u(y(\theta)) \) is increasing in \( e \). From the envelope theorem, the fact that \( f(L, \cdot) \) is increasing, and the definition of stochastic dominance, \( \pi(\cdot) \) is increasing in \( e \). Now, with \( u(y) = \ln y \), \( \sum h(e, \theta)/u(y(\theta)) = \sum h(e, \theta)y(\theta) = \pi(e) + a \) which is increasing in \( e \), as desired.

References


