Bulletin of Glaciological Research 29 (2011) *19–29*

Article

On Characteristic Timescales of Glacier AX010 in the Nepalese Himalaya

Surendra ADHIKARI[1], Shawn J. MARSHALL[1] and Philippe HUYBRECHTS[2]

1 Department of Geography, University of Calgary, 2500 University Drive NW, Calgary, AB, T2N 1N4, Canada
surendra.adhikari@ucalgary.ca

2 Earth System Sciences and Department of Geography, Vrije Universiteit Brussel, Pleinlaan 2, B-1050 Brussels, Belgium

(Received September 21, 2010; Revised manuscript received March 23, 2011)

Abstract

Observations indicate that over the past few decades, valley glaciers and ice caps in the Nepalese Himalaya have been continuously shrinking in response to climate warming. The response timescales of these glaciers are not yet well understood. Considering the case of Glacier AX010, this paper examines several methods for estimating the rate of glacier response to changes in climate. In spite of having simple model physics and requiring only a few and often available field data, simpler analytical methods yield reasonable estimates of timescale. Detailed analytical and numerical ice-flow models suggest that the response times for Glacier AX010 are on the order of 50 years. These magnitudes are slightly larger than field evidence indicates for typical valley glaciers, indicating that Glacier AX010 responds relatively slowly to changing climate. Nonetheless, sustained century-scale warming as forecasted for the Nepalese Himalaya would provide sufficient time for Glacier AX010 to respond to this climatic disequilibrium. Given the fact that the glacier already lacks a persistent accumulation zone, we foresee a complete retreat of the glacier by 2100.

Key words: reaction time, response time, Glacier AX010, valley glacier, Nepalese Himalaya

1. Introduction

Meteorological records indicate that the Nepalese Himalaya warmed in the late 20th century (Shrestha *et al.*, 1999), accompanied by near-constant or decreasing precipitation (Shrestha *et al.*, 2000). The impacts of such climate change on glacial systems are observed in terms of shrinkage of areal extent (*e.g.*, Fujita *et al.*, 2001), loss of ice volume, and the formation and drainage of glacial lakes (Kattelmann, 2003). Over the past several decades, this glacier retreat and hydrological response have caused severe damage to lives and properties downstream of many Himalayan glaciers (*e.g.*, Richardson and Reynolds, 2000). The situation is expected to worsen in the future, as climatic projections follow similar trends until at least the end of the 21st century (Christensen *et al.*, 2007). In an attempt to understand the response of a typical Nepalese Himalayan glacier to climate change, this paper assesses several available methods to estimate (1) how quickly the glacier length responds to a climatic shift, and (2) how long the glacier takes to adjust its overall geometry following a sustained change in climate.

In response to a climatic shift, glaciers adjust their geometry in all three dimensions. Glacier thickness responds immediately to perturbations in snow accumulation and/or snow/ice ablation, but changes in glacier extent (area or length) become noticeable only after some delay. This lag in time, typically on the order of a few years, is generally referred to as the reaction time (*e.g.*, Pelto and Hedlund, 2001). The reaction time can be estimated from the timescale for a switch in direction of geometric adjustment (retreat to advance, or vice versa) following multi-year or decadal-scale climate variability. Identifying the reaction time for a glacier undergoing a continuous retreat or advance is difficult. Nevertheless, statistical analysis between climatic data and corresponding glacier extent data (*e.g.*, McClung and Armstrong, 1993; Calmanti *et al.*, 2007) can yield an estimate of this timescale.

Glacier response time is a clearly-defined and widely-used timescale to characterize a glacier's response to climate change. It is broadly defined as the period over which a glacier undergoes a geometric adjustment to accommodate a change in climate. Response times for both ice volume and glacier length (extent) can be considered. These timescales provide useful information about how quickly a glacier evolves from one to another equilibrium state and the length of time that a glacier is affected by a climatic shift, *i.e.* how long a glacier carries the climate history in its memory.

Based on both field observations and numerical/empirical models, several methods (*e.g.*, Jóhannesson *et al.*, 1989a, b; Harrison *et al.*, 2001; Oerlemans, 2001; Raper and Braithwaite, 2009) have been proposed to estimate the response time. The majority of these methods provide estimates on the order of a few decades for the response time of valley glaciers, compared with centuries to millennia for larger icefields and ice sheets. This rapid response of valley glaciers is mainly due to the typical mountain climate, rather than the dynamic characteristics of the glacier arising from its small size (Bahr *et al.*, 1998). This paper explores several methods to assess their ability to estimate response times for a real valley glacier. We discuss the strengths and limitations of each method.

2. Study Area

Due to its simple geometry, absence of an icefall, and lack of debris cover, Glacier AX010 (Shorong Himal; 27°42′N, 86°34′E; Figure 1) is treated here for the diagnosis of its response timescales. As of AD 1978, the glacier was 1.7 km long with an areal extent of 0.57 km^2 and an altitudinal range of 4952–5360 m a.s.l. The glacier flows eastward in its accumulation zone and its terminus runs down to the southeast. The planform geometry of the glacier is typical V-shape, with the accumulation area about four times wider than the ablation zone. It is a monsoon-affected summer-

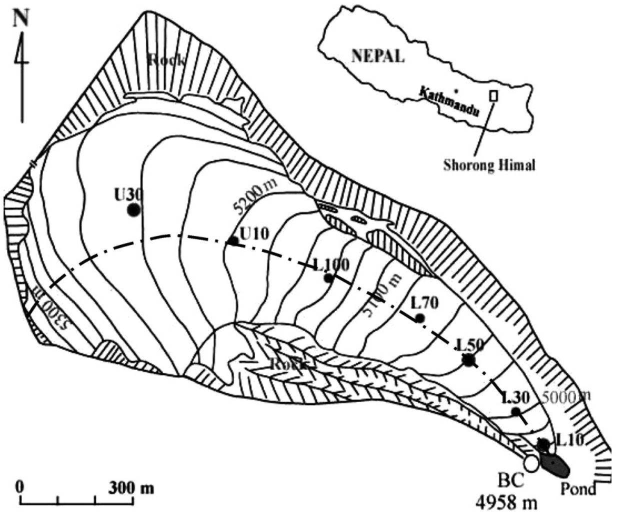

Fig. 1. Location of Shorong Himal and topographic map of Glacier AX010 as surveyed in AD 1978 (Ikegami and Ageta, 1991). Solid circles inside the glacier are positions of mass balance stakes. Solid lines across the glacier are surface elevation contours of 20 m interval. The dotted line along the glacier is the assumed flow line used in the numerical ice-flow model.

accumulation-type glacier (*e.g.*, Kadota and Ageta, 1992). The primary accumulation and ablation period coincide, and summer-balance is representative of the annual mass balance.

Glacier AX010 is amongst a few highly-studied glaciers in the Nepalese Himalaya. The first detailed study of this glacier was conducted in 1978. In this year, mass balance (Ageta *et al.*, 1980), heat balance (Ohata and Higuchi, 1980), surface velocity (Ikegami and Ageta, 1991), and areal extent data (WGMS, 1998) were collected. Thereafter the glaciological state variables have been monitored intermittently in 1989, 1991, 1995–1999, and 2004. Only the terminus position was recorded in 1989 (Fujita *et al.*, 2001), while in 1991 topographic mapping of the whole area of the glacier was completed (WGMS, 1998). Annual monitoring of the glacier was initiated in 1995 and continued until 1999 to obtain mass balance, surface velocity, and areal extent data (WGMS, 2005). The ice thickness was measured in 1995 (Kadota *et al.*, 1997) by means of radio-echo sounding. The glacier extent was also resurveyed in 2004 (Kayastha and Harrison, 2008).

3. Reaction Time

For any glacier there exists a time lag between the onset of a relatively sudden change in climate and the initiation of a noticeable response of its terminus. This lag in time for initial terminus response is referred to as the reaction time τ_r. Unlike the response time, this timescale is not a pure physical property of a glacier (*e.g.*, Oerlemans, 2001; Pelto and Hedlund, 2001), as it depends not only on the state of glacier (the degree to which it approximates an equilibrium state), but also on the climate history. Furthermore, the term 'noticeable response' appearing in the definition can be ambiguous; for instance, glacier margins often undergo annual advance/retreat cycles. Therefore the reaction time has been criticized as a loosely-defined term. Nevertheless there is an intrinsic value to this metric, as terminus fluctuations are an intuitive and readily observable feature of valley glaciers. Here we investigate τ_r for Glacier AX010 through a statistical correlation between historical climate variations and resulting position of the glacier terminus.

We consider temperature as the key climatic parameter that determines the mass budget of Glacier AX010, as done in several other studies (*e.g.*, Naito *et al.*, 2001; Adhikari and Huybrechts, 2009). Due to the unavailability of a sufficiently long series of temperature recorded on or nearby the glacier, we use data from the Kathmandu station (~150 km west of the glacier). Kayastha and Harrison (2008) infer that Kathmandu temperature is highly correlated to regional variations in Equilibrium Line Altitude (ELA) in the eastern part of the Nepalese Himalaya. Kathmandu summer temperature also follows a similar trend of variation as the corresponding data from the Chialsa station, which is located ~20 km south of the glacier and has data from 1976 to 1996. Naito *et al.* (2001) use Chialsa data in their research on Glacier AX010. Data from Chialsa and Kathmandu are highly correlated for the period of overlap (r=0.81). Adhikari and Huybrechts (2009) successfully reconstruct the historical terminus position of Glacier AX010 by forcing an ice-flow model with temperature anomalies from Kathmandu station.

Observations of terminus position are available for Glacier AX010 (Figure 2a). The lack of switch in direction of glacier length changes (from retreat to advance, or vice versa) makes it difficult to judge τ_r based on the visual inspection of temperature trend and the glacier terminus positions as in Pelto and Hedlund (2001). While there are only sparse measurements of glacier terminus position (9 observations over 26 years, from 1978 to 2004), we are able to perform a cross-correlation analysis between time series of climate and glacier extent data (Figure 2a) in order to measure the degree of their linear relationship. Similar techniques have been used to estimate τ_r for Blue Glacier, USA (McClung and Armstrong, 1993) and for glacial systems in Piedmont and Val d'Aosta, Italy (Calmanti *et al.*, 2007).

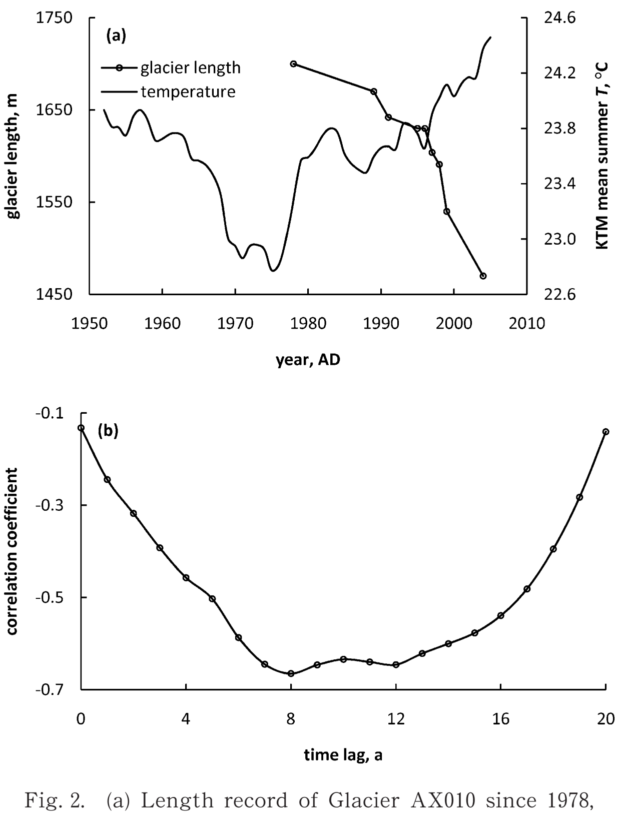

Fig. 2. (a) Length record of Glacier AX010 since 1978, along with a 3-year running mean of Kathmandu (KTM) summer (June-July-August-September) temperature. (b) Cross-correlation analysis between mean summer temperature and the glacier length over the period 1978–2004. A positive lag is applied to the length data.

We apply a time lag of 0 to +20 years to the length series by shifting its position backward in time and plot the correlation coefficient between the temperature and glacier length for each length of the lag (Figure 2b). The figure reveals good correlations for time lags of 7–12 years (with the peak occurring at 8 years, r=−0.67), implying that it takes about 8 years for Glacier AX010 to initiate its terminus retreat in response to the applied negative mass balance (as a result of a warming climate). This estimate (τ_r=8 years) is based on the assumption that the glacier retreated linearly between the measurement years. This magnitude of reaction time is comparable to those reported for other small valley glaciers (τ_r=4–16 years; McClung and Armstrong, 1993; Pelto and Hedlund, 2001; Calmanti *et al.*, 2007).

4. Response Time

The generally accepted definition of response time is based on the concept of an equilibrium state glacier. A glacier is said to be in an equilibrium state when its dimensions remain fixed under a constant mass balance. A step change in mass balance on such an idealized glacier induces a response towards a new equilibrium state. The time that a glacier takes to move from the initial to a new equilibrium state is precisely defined as the equilibrium time (Bahr *et al.*, 1998). In principle, this timescale can be infinitely long. It is therefore common to use the e-folding time as the characteristic measure of the response time (Jóhannesson *et al.*, 1989b): the time required for a glacier to achieve $(1-e^{-1})\approx 63\%$ of the total geometric change along an exponential, asymptotic path to a new equilibrium state.

An idealized step change in climate causing an exponential, asymptotic evolution of glacier from one to another equilibrium state does not occur in nature. So, the suitability of response times estimated from field data relies on, for example (1) how far the initial glacier geometry is off the equilibrium state, (2) how well the climate variability represents a step change, (3) whether the study period is long enough for a near-complete adjustment in glacier geometry, and (4) how accurately the observed field data represent the input variables needed to estimate response time. With careful consideration of these issues, we evaluate response times for Glacier AX010. The suitability of estimated values is discussed based on the physics and constraints of employed models.

4.1 Simpler analytical approaches

The Nye method

Based on the theory of propagation and diffusion of kinematic waves (Nye, 1960 and a subsequent series of his papers), a semi-quantitative method is proposed to estimate a memory timescale of glacier. This timescale is equivalent to the volume timescale τ_v (Jóhannesson *et al.*, 1989b), which is proportional to the length L and inverse of terminus velocity u_t of an (initial) equilibrium state glacier:

$$\tau_v = f\frac{L}{u_t}\,. \tag{1}$$

Here f is the time-dependent proportionality factor that explains along-flow evolution of changes in surface elevation following a step change in climate. Given the time, it is formally defined as a ratio of thickness change averaged over the full glacier length $\overline{\Delta H}$ to the change at its terminus ΔH_t :

$$f = \frac{\overline{\Delta H}}{\Delta H_t}\,. \tag{2}$$

In order to estimate τ_v, this factor f should correspond to the value obtained after a sufficiently long time over which a glacier completes the majority of its geometric adjustment.

Now we compute τ_v for Glacier AX010. Continuous retreat of the glacier (Figure 2a) indicates that it was not in an equilibrium or near-equilibrium state over the period 1978–2004. We choose the glacier in 1978 as an initial domain because good model baseline data are available for this year. The glacier length and average terminus velocity (observed at L10, Figure 1) were 1700 m and 4.0 m a^{-1} respectively (Ikegami and Ageta, 1991). The thickness data required to calculate the factor f are listed in Table 1. The glacier-average thickness change over 21 years (1978–1999) was about −17 m. The change at the terminus during 1978–1991 was about −30 m (Kadota *et al.*, 1993). For 102 m of glacier retreat over 1991–1999, a simple geometric analysis (Kadota and Ageta, 1992) with a smooth surface slope of about 15° (as revealed by 1991 topographic map) yields a 27 m lowering of glacier surface at the terminus. With ±10 m of accuracy, these data result in $f \approx 0.30 \pm 0.06$ (1978–1999 average). These values of f fall in the range of 0.1–0.4 as calculated by Schwitter and Raymond (1993) using the along-flow profile change data of 15 glaciers.

The values $L = 1700$ m, $u_t = 4.0$ m a^{-1}, and $f = 0.30 \pm 0.06$ yield τ_v in the range between 102 and 153 years (126 years for $f = 0.30$). It is apparent that a slight change in factor f causes a large change in response time. The model is equally sensitive to terminus velocity, which is often spatially variable.

The Nye model is primarily designed for a parallel-sided slab with unlimited lateral extent. Its estimates may not be representative of valley glaciers which have finite and varying width. In order to address this issue, we recall Jóhannesson *et al.* (1989b) to note that τ_v in Eq. 1 indicates the time needed for ice to traverse the full glacier length if it were to move the complete distance at the speed of the terminus. The plan-form geometry with the wide accumulation and narrow ablation zones (Figure 1) indicates converging flow on Glacier AX010. Consequently ice may traverse downstream at a faster speed than the terminus speed. It might therefore be useful to estimate τ_v using the velocity near the ELA. The velocity at ELA (∼5200 m a.s.l.) in 1978 was about 6.7 m a^{-1} (point U10 in Figure 1). For $f = 0.3$, this gives $\tau_v = 76$ years.

The Jóhannesson method

Jóhannesson *et al.* (1989a) propose an equally sim ple approach to estimate τ_v. According to this method, the timescale is obtained by dividing a characteristic ice thickness H by the net annual mass balance (ice-equivalent) at the glacier terminus b_t:

$$\tau_v = \frac{H}{-b_t}\,. \tag{3}$$

For a parallel-sided glacier resting on relatively smooth bedrock, the characteristic thickness represents the maximum ice thickness. If a glacier has undulating subglacial topography and varying lateral extent, H should be adjusted properly. In spite of having irregular basal topography and varying width, maximum ice thickness has been used for several valley-glacier applications (*e.g.*, Schwitter and Raymond, 1993; Naito *et al.*, 2001; Pelto and Hedlund, 2001). For typical cases, effects of basal topography and varying glacier width might cancel each other. We shall consider this point in Section 4.5.

Kadota *et al.* (1997) measured ice thickness of 86, 83 and 51 m at three different locations along the central flow line of Glacier AX010. By preserving these data, Adhikari and Huybrechts (2009) approximate the bedrock topography using a simple method (Nye, 1952) to estimate ice thickness:

$$H = \frac{S_d}{\rho g}\left|\frac{dh}{dx}\right|^{-1}. \tag{4}$$

Table 1. Record of glacier thickness change.

time period	glacier-average, m	at terminus, m	reference
1978-1991	−8.69†	−30‡	†WGMS (1998), ‡Kadota et al. (1993)
1991-1996	−5.70	-	Kayastha and Harrison (2008)
1996-1999	−2.47	-	WGMS (2005)

Here S_d is the gravitational driving stress, ρ is ice density, g is gravitational acceleration, and h is surface elevation. For 1996, this suggests maximum value of ice thickness to be around 120 m. The net annual mass balance values recorded at the glacier terminus from 1996 to 1999 were −1.72, −2.58, −3.60, and −2.91 m w.e. a^{-1} (WGMS, 2005), giving an annual average of $b_t = -2.97$ m (ice equivalent) a^{-1}. Eq. 3 yields τ_v to be about 40 years. This value is comparable to the corresponding timescales of many other glaciers ($\tau_v = 6-60$ years; McClung and Armstrong, 1993; Paterson, 1994; Oerlemans, 1997, 2001; Pelto and Hedlund, 2001).

Discussion

A quick comparison of τ_v for Glacier AX010 obtained from the methods discussed above reveals a broad range of magnitude. Both models are based on simple physics and are designed for glaciers with simple geometry. They are equally sensitive to model input. Although this is often misinterpreted, both of these models are insensitive to details of ter minus dynamics†. However, the Jóhannesson method appears to yield timescale (*e.g.*, $\tau_v = 40$ years for Glacier AX010), which is more typical of valley glaciers (*e.g.*, Jóhannesson *et al.*, 1989b; Paterson, 1994; Oerlemans, 2001).

Despite the appealing simplicity and practical utility of these methods, they bear an additional degree of uncertainty in that none of them accounts for some of the key phenomena of glacier-climate interaction, such as the altitude - mass balance feedback. This feedback is crucial as accumulation and ablation rates are dependent on glacier surface elevation. Analytical solutions reveal that inclusion of such feedbacks may double or triple the response time (Oerlemans, 2001).

4.2 Including the altitude - mass balance feedback

To date, only a few attempts have been made to include the altitude - mass balance feedback in analytical models. They include a proposal by Harrison *et al.* (2001) to modify the Jóhannesson estimate (Eq. 3), a similar effort by Oerlemans (2001), and a simple conceptual model of glacier hypsometry by Raper and Braithwaite (2009). The applications of these models are not straightforward as they require extensive data on both climatic and geometric details. Here we consider the Harrison model to demonstrate the importance of including the mass-balance dependency on glacier surface elevation.

The Harrison model introduces the idea of a reference glacier geometry, which can be taken as the initially-surveyed volume V_0, area A_0, and surface topography $z_0(x, y)$ of an ice mass. Over time the glacier evolves to a new geometry $V(t)$, $A(t)$, and $z(x, y, t)$, with the changes ΔV, ΔA, and $\Delta z(x, y)$ respectively. Based on the concept of a reference surface balance rate (Elsberg *et al.*, 2001), Harrison *et al.* (2001) modify the Jóhannesson model so that:

$$\tau_v = \frac{1}{\left(\frac{-\overline{b}_r}{H} - \overline{G}\right)} . \tag{5}$$

Here H is once again the characteristic ice thickness, b_r is the specific balance rate at the (ice-free) bedrock surface, and $\overline{b}_r$ is the average value of this quantity over area ΔA:

$$\overline{b}_r \sim \frac{\int_{\Delta A} b_r \, dA}{\int_{\Delta A} dA} . \tag{6}$$

This accounts for the effect of changes in glacier area ΔA on the balance rate. Similarly, $\overline{G}$ is the area-weighted average of vertical gradient in specific mass balance rate $G = \dfrac{db}{dz}$:

$$\overline{G} \sim \frac{\int_{A_0} \mathrm{G}(\mathrm{z} - \mathrm{z}_0)\, dA}{\int_{A_0} (z - z_0)\, dA} . \tag{7}$$

The $\overline{G}$ term accounts for the effect of changing surface elevation on the glacier mass balance rate, where $(z - z_0)$ is the difference in elevation at a point on the surface with respect to the reference surface.

The τ_v defined by Jóhannesson *et al.* (1989a, Eq. 3) and modified by Harrison *et al.* (2001, Eq. 5) have a similar physical basis except that the latter one accounts explicitly for the altitude - mass balance feedback via $\overline{G}$. Moreover b_t in Eq. 3 characterizes the balance rate at the elevation of the ice surface near the terminus, while $\overline{b}_r$ in Eq. 5 characterizes the weighted-average of the balance rate at bedrock elevation over ΔA. Where a retreating glacier experiences ice loss all around its perimeter (*i.e.* at both low and high elevations), the absolute magnitude of $\overline{b}_r$, will be less than that of b_t. The opposite is likely to be true where ΔA is dominated by glacier losses at the glacier front, as the newly exposed bedrock lies at lower elevations than the current terminus. It is therefore hard to make generalization about the relationship between $\overline{b}_r$ and b_t. We consider $\overline{b}_r = b_t$ as in Leysinger Vieli and Gudmundsson (2004), because it is difficult to accurately determine $\overline{b}_r$ as it demands a map of bedrock elevations within ΔA and the specific balance rates b_r at those elevations. Such details of mass balance data are not available for the glacier at hand.

We consider the surface of Glacier AX010 in 1978 (Figure 1) as the initial surface, *i.e.* at time $t = 0$ years. Although it is recommended to estimate $\overline{G}$ for each

† Although both f and u_t in the Nye model are individually dependent on the dynamical processes local to the terminus zone, their ratio and hence τ_v is broadly insensitive to details of terminus dynamics (Jóhannesson *et al.*, 1989b). The physics of Jóhannesson model does not rely on such dynamics.

Table 2. Summary of various parameters used to calculate $\overline{G}$. The value of G is computed using mass balance in elevation ranges one up and one below. For the first and the last elevation ranges, it is obtained by linearly extrapolating adjacent three values.

elevation, m a.s.l.		A_0	$(z - z_0)$	b	$G = db/dz$	$G(z - z_0)\,dA$
range	mean	m^2	m	$m\ a^{-1}$	$m\ a^{-1}\ m^{-1}$	$m^3\ a^{-1}$
5250-5360	5305	174500	−1.8	0.15	0.0047	−1476.27
5200-5250	5225	172500	−6.3	−0.12	0.0066	−7189.27
5150-5200	5175	68500	−11.2	−0.71	0.0118	−9052.96
5100-5150	5125	40500	−15.3	−1.30	0.0161	−9976.37
5050-5100	5075	61500	−18.4	−2.32	0.0135	−15276.60
5000-5050	5025	40500	−21.2	−2.65	0.0066	−5637.27
4952-5000	4976	11000	−20.2	−2.97	0.0048	−1066.56
		$\int_{A_0} G(z - z_0)\,dA$, $m^3\ a^{-1}$				−49675.3
		ΔV, m^3				−5000100.0
		$\overline{G}$, m w.e. $a^{-1}\ m^{-1}$				0.00993

year and to perform a numerical integration, this is not possible for Glacier AX010, due to the unavailability of annual hypsometry, mass balance, and thickness change data. We therefore estimate $\overline{G}$ for the period 1978–1991 in a single treatment. We obtain hypsometry and thickness change data from WGMS (1998). We assume the balance rate gradient of the conventional surface is equivalent to the reference surface counterpart (Elsberg *et al.*, 2001) and use the average annual data recorded during 1996–1999. Data and parameters considered to estimate $\overline{G}$ for Glacier AX010 are summarized in Table 2. The calculation yields $\overline{G}=9.9$ mm (ice-equivalent) $a^{-1}\ m^{-1}$ (1978–1991 average).

The third input parameter is characteristic ice thickness, which is defined as the derivative of volume with respect to area. Due to the lack of essential field data, we obtain $H=\Delta V/\Delta A$ using a dynamical ice-flow model (Section 4.3). On the course of simulating the historical front position (Adhikari and Huybrechts, 2009), we obtain the initial geometry (AD 1978) of Glacier AX010 with $A_0=0.57\times10^6\ m^2$ (both observed (WGMS, 1998) and simulated) and $V_0=26.76\times10^6\ m^3$ (simulated). For each year thereafter, we obtain annual data for surface area and ice volume. Ice-equivalent cumulative change in volume ΔV and area ΔA with respect to their initial values are shown in Figure 3. This gives $H=117$ m, which is comparable to the maximum ice thickness (120 m), as in the case with South Cascade glacier (Harrison *et al.*, 2001). The resulting response time from Eq. 5 is 69 years. Comparing this value to the Jóhannesson estimate (40 years) illustrates the importance of the altitude - mass balance feedback. Including this feedback increases the response time by 72%. The sensitivity of response time to the mass balance gradient is also assessed. Switching $\overline{G}$ by ±2 mm (ice-equivalent) $a^{-1}\ m^{-1}$ causes the response time to vary by about 9 years on average. As evident from Eq. 5, longer timescale is obtained when there is stronger mass balance gradient ($\overline{G}=11.9$ mm of ice-equivalent $a^{-1}\ m^{-1}$ in our tests).

4.3 *Calculations with a numerical ice-flow model*

In this section, we obtain both volume and length response times for Glacier AX010 by using a dynamical ice-flow model. As Leysinger Vieli and Gudmundsson (2004) suggest that simpler models such as those based on the shear-deformational flow yield

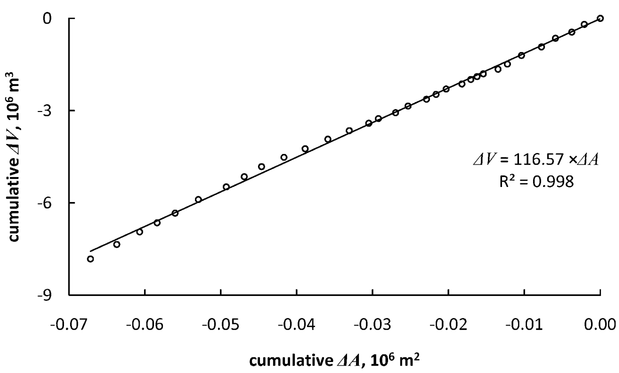

Fig. 3. Ice-equivalent cumulative change in volume ΔV as a function of change in area ΔA of Glacier AX010, obtained from a dynamical ice-flow model. Magnitudes of ΔV and ΔA are relative to the initial glacier geometry (AD 1978). The slope of the linear fit gives the Harrison thickness scale.

sufficiently accurate timescales, we consider a simple model rather than a high-order or Stokes treatment of glacier dynamics. The prognostic equation of our model links change in ice thickness H to the flux divergence (first term in the RHS) and the net surface mass balance b:

$$\frac{\partial H}{\partial t}=-\frac{1}{(w_0+2H)}\ \frac{\partial\,[HU(w_0+H)]}{\partial x}+b\,. \qquad (8)$$

Here w_0 is bottom width of the glacier, x is distance along the flow line, t is time, and U is the vertically-averaged velocity. The velocity is comprised of deformational and sliding components, $U_d=f_d HS_d^3$ and $U_s=f_s S_d^3 H^{-1}$, where f_d and f_s are flow parameters and $S_d=-\rho gH\dfrac{dh}{dx}$ is the gravitational driving stress. Details on model set-up and parameter description can be found in Adhikari and Huybrechts (2009).

The climatic setting imposed in the model is simple and can be described using a linear mass balance gradient of 0.01 m w.e. a^{-1} m^{-1} (*e.g.*, Harper and Humphrey, 2003):

$$b=0.01(h-5174)+\Delta b\,. \qquad (9)$$

The dependence of mass balance on glacier surface h captures the altitude - mass balance feedback. An additional perturbation term Δb (constant in time and elevation) is included to test the climatic response of the glacier. We impose a step change in mass balance in a range between $\Delta b=-0.5$ and $\Delta b=+0.5$ m w.e. a^{-1} on the equilibrium state glacier whose longitudinal extent resembles the one observed in 1996. For each case, the resulting evolutions of glacier length and ice volume are shown in Figure 4. This gives e-folding timescales of $\tau_l=55$–86 and $\tau_v=48$–64 years to adjust glacier length and ice volume, respectively. These values are comparable to response times for several other valley glaciers ($\tau_l=20$–114 years and $\tau_v=13$–73 years; *e.g.*, Huybrechts *et al.*, 1989; Oerlemans, 1997, 2001; De Smedt and Pattyn, 2003).

Based on the reaction of glacier length (Figure 4a) and ice volume (Figure 4b), there are two interesting points to note. First, Glacier AX010 takes less time (by 3–26 years) to adjust its ice volume than its length, due to the instantaneous response of ice thickness to the climatic perturbation. Second, response times are shorter for a severe climate change ($\Delta b=\pm0.5$ m w.e. a^{-1}) than for a mild one ($\Delta b=\pm0.1$ m w.e. a^{-1}); τ_v and τ_l (tau−1) are reduced in these cases by 12 and 20 years, respectively. For larger perturbations of the mass balance, the response of the glacier is more non-linear (*e.g.*, Jóhannesson *et al.*, 1989b), thereby inciting a more rapid initial response. This yields smaller e-folding timescales.

4.4 *Calculations based on the glacier length records*

Here we attempt to calculate the response time τ_l of Glacier AX010 from its length record. For an equilibrium state glacier with length L_0 and ELA E_0, Oerlemans (2001) relates the fluctuation in ELA ΔE to change in glacier length ΔL as:

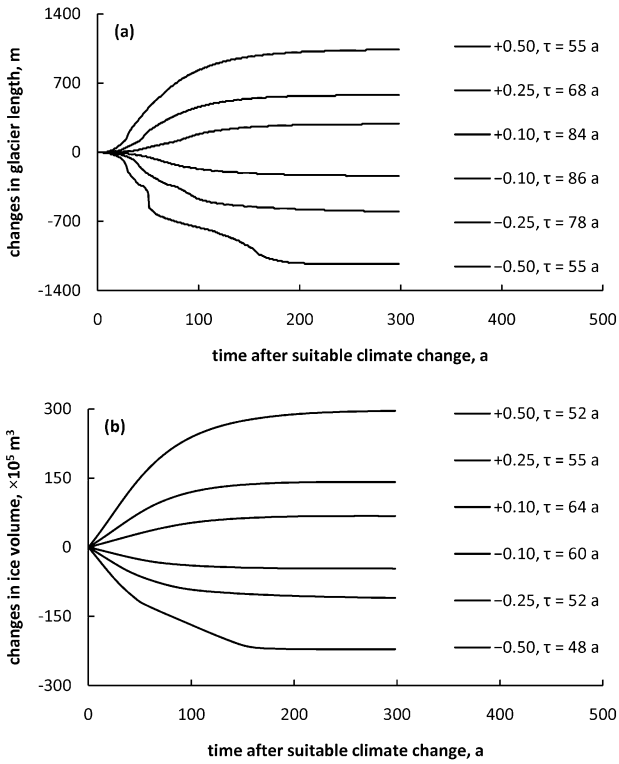

Fig. 4. Reaction of the glacier length (a) and volume (b) in response to applied changes in mass balance, obtained from a dynamical ice-flow model. In both cases, a step change in mass balance in the range between −0.5 and +0.5 m w.e. a^{-1} is imposed on an equilibrium state glacier whose longitudinal extent is equivalent to the 1996 glacier stand.

$$\Delta E(t)=\frac{1}{c}\Big[\Delta L(t)+\tau_l\frac{d}{dt}\Delta L(t)\Big]\,. \qquad (10)$$

Here $L(t)=L_0+\Delta L(t)$ is the glacier length and $E(t)=E_0+\Delta E(t)$ is the ELA at any time t, and $c<0$ is the climate sensitivity that determines how the equilibrium glacier length is related to the ELA.

First, we linearly interpolate sparsely observed glacier length data (Figure 2a) and prepare its annual record for the period 1978–2004. It is fitted with a fourth-order polynomial (Figure 5a). Given the time t, the change in length $\Delta L(t)$ and the rate of change in length $\dfrac{d}{dt}\Delta L(t)$ are obtained from the polynomial and its first derivative, respectively. Next, we investigate the climate sensitivity using the dynamical ice-flow model discussed in Section 4.3. ELA perturbations in a range between −25 and +25 m are imposed on the equilibrium state glacier as defined in the previous section in order to obtain the corresponding changes in glacier length. The results are plotted and fitted with a least-square linear line (Figure 5b), whose slope determines the climate sensitivity ($c=-24.6$) for Glacier AX010. As the figure reveals a highly linear relation between $\varDelta E$ and $\varDelta L$, the linear inverse model should yield a reasonably accurate estimate of τ_l.

In order to compute the length response time, we now require an annual record of ELA. Glacier AX010 lacks such data. For the Rolwaling massif that hosts Glacier AX010, Kayastha and Harrison (2008) obtain $\varDelta E = 1.00 \pm 0.88$ m a^{-1}. The upper limit of recommended $\varDelta E$ seems to represent well the conditions for Glacier AX010, because the glacier has experienced an increasingly warmer climate and has even lost its accumulation zone for some years during the study period (*e.g.*, WGMS, 2005). Nevertheless, we consider both average ($\varDelta E = 1.00$ m a^{-1}) and maximum ($\varDelta E = 1.88$ m a^{-1}) values. We now reconstruct the ELA by considering τ_l as a tuning parameter in the model (Eq. 10), such that the linear trend of reconstructed ELA best describes the employed $\varDelta E$. A typical reconstruction of $\varDelta E(t)$ is shown in Figure 5a. Corresponding response times for mean and maximum $\varDelta E$ are found to be 22 and 51 years, respectively. These timescales along with climate sensitivity are comparable to corresponding values for several other glaciers ($c = 8$–84 and $\tau_l = 4$–87 years; Oerlemans, 2001, 2007; Klok and Oerlemans, 2003).

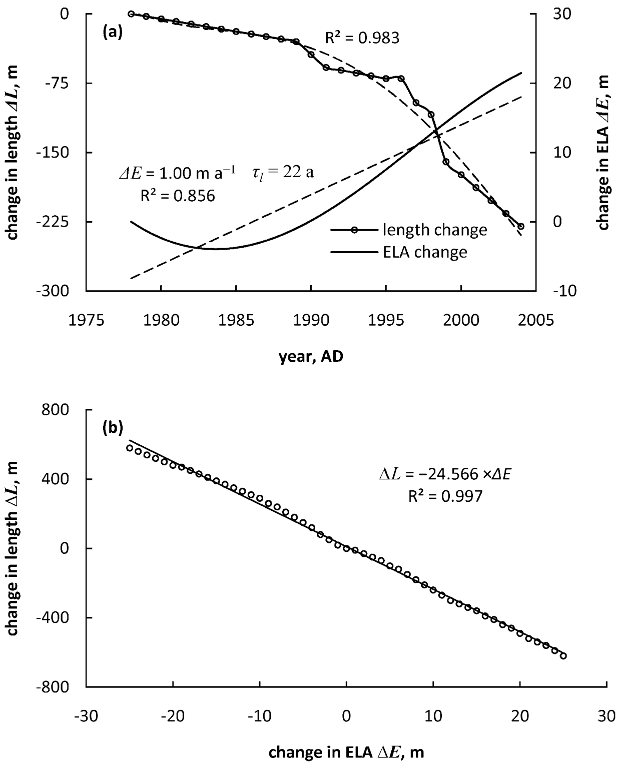

Fig. 5. (a) Glacier length and reconstructed ELA obtained from the inverse model. Both curves are plotted with respect to 1978 data, and are accompanied by the corresponding trend lines. The trend line fitted over the length record is a polynomial of degree $N = 4$, while that over the ELA is a linear fit. (b) Resulting changes in an equilibrium state glacier length due to applied changes in the ELA. The slope of the linear fit reveals the glacier's climate sensitivity.

4.5 *Discussion*

It is constructive to synthesize and compare the timescales obtained from several approaches. This paper deals with three analytical methods, one numerical ice-flow model and one linear inverse model. With the ability of numerical flow model to yield both volume and length response times, we obtain τ_v from four and τ_l from two different methods (Table 3). Summing up the key strengths and limitations of each method, we discuss the suitability of estimated timescales to represent Glacier AX010 in broader sense.

Volume response time

Simpler analytical methods are easy to use as they require very little field data, which is commonly available. However these models contain simplified underlying physics and are associated with numerous constraints. The Nye model is based on a linearized treatment of ice motion and is applied to simple glacier geometry. Estimates from this model may not be suitable, because (1) the climatic perturbation (scaled to a step-change) over the study period could be large enough that non-linear effects are important, and cannot be captured via linear theory, (2) the glacier in 1978 could be well off the equilibrium state, and

Table 3. Response times for Glacier AX010 as obtained from several methods.

description of the method		τ_v, a	τ_l, a
analytical methods	the Nye model	126, 76†	-
	the Jóhannesson model	40	-
	the Harrison model	69	-
numerical ice-flow model	-	48-64	55-86
linear inverse model	-	-	22, 51‡

† For the velocity at terminus and ELA, respectively.

‡ For the mean and maximum rate of recommended change in ELA, respectively.

(3) the applied velocities may not represent the width-averaged values. As explained in Section 4.1, the first estimate (τ_v=126 years) is longer than it should be, because it does not capture the effect of converging flow caused by varying glacier width. We have more confidence in the result that is based on velocity around the ELA (τ_v=76 years). The use of the velocity at the ELA, however, is neither theoretically justified nor commonly practiced.

Compared to the Nye estimates, the Jóhannesson estimate (τ_v=40 years) is more typical of valley glaciers. However, its reliability is dependent on how well the ice thickness value represents the effects of irregular basal topography and varying glacier width. The presence of subglacial deep basins (see Figure 3 in Adhikari and Huybrechts, 2009) provides an extra resistance (compared to smooth bedrock) and impedes the ice flux, thereby forcing the glacier to respond to climate change gradually (*i.e.*, longer timescale). In convergent flow associated with varying glacier width (Figure 1), on the other hand, ice tends to flow faster, thereby accommodating the change in climate more rapidly (*i.e.* shorter timescale). Should these effects compensate for each other, the Jóhannesson estimate would be more reliable. The ice-thickness scale as obtained from a dynamical ice-flow model that accounts for the effects of irregular basal topography and varying width of Glacier AX010 (see Section 4.2, H=117 m) is indeed consistent with the thick ness used in Eq. 3 (H=120 m). This justifies the choice of characteristic thickness and hence the reliability of τ_v.

Besides several constraints pointed above, simple analytical models do not account an important feedback between surface elevation and mass balance. Including such a feedback rigorously demands extensive annual data concerning both glacier geometry and climate. By making use of available data for Glacier AX010, the Harrison model accounts for the altitude - mass balance feedback and yields τ_v=69 years. This is generally an improved estimate of the Jóhannesson model, but its reliability depends on (1) how well the balance rates observed at the ice surface near the terminus and at the bedrock elevation match each other, and (2) whether the assumption of similar balance rates for conventional and reference glacier surfaces is valid.

Although it only simulates shear-deformational flow, the employed numerical model deals with more comprehensive (non-linear) dynamics of glacier ice-flow than the analytical methods. Furthermore, the numerical model accounts for the effects of varying glacier width and altitude - mass balance feedback. Therefore these estimate (τ_v=48–64 years) are likely to be more realistic and representative for Glacier AX010. In the ongoing climatic context (rapid warming), the lower estimates (on the order of 50 years), which are associated with larger perturbations in mass balance might represent the most suitable volume response time for Glacier AX010.

Length response time

Although the volume timescale is of much interest from a practical point of view (*e.g.*, for water resources studies or issues of sea level change), estimates of τ_l that explain the rate of adjustment of glacier length are equally useful. With explicit treatment of flow dynamics, consideration of evolving glacier geometry, and inclusion of the altitude - mass balance feedback, the numerical ice-flow model yields generally acceptable and possibly representative timescale (τ_l=55–86 years). In the ongoing climatic context with increasingly warmer conditions, the lower estimates (on the order of 50 years) associated with the larger perturbations in mass balance should represent the most suitable length response time for Glacier AX010. This is consistent with the inverse model estimate (τ_l=51 years) for a larger rate of change in ELA (ΔE=1.88 m a^{-1}). This estimate, however, is based on simple physics that linearly relates ELA fluctuations to a change in glacier length, and is prone to errors associated with sparse measurements of glacier length over the study period.

5. Conclusion

In an attempt to quantify the climatic response of Glacier AX010, this paper considers several approaches to obtain suitable values of its characteristic timescales. First, we compute the reaction time using a simple statistical method, in order to understand how quickly the glacier exhibits its initial terminus response to ongoing climate change. Although we lack annual observations of glacier length, the available data and analysis are sufficient to estimate a reaction time of about 8 years for Glacier AX010.

Next, we estimate the response time using a number of analytical and numerical models, in order to quantify how long the glacier takes to adjust its overall geometry following an observed or idealized change in climate. Unsurprisingly, each method yields a unique result, thereby producing a broad range of estimates. It is therefore difficult to report a definitive value for the response time. Based on considerations of the physics and constraints of each models, the accuracy and consistency of input parameters, and the ongoing (increasingly warmer) climatic context, we conclude that representative values of response time (both volume and length) for Glacier AX010 are about 50 years. This indicates a relatively slow response of the glacier to climate change, as generally recommended values for small valley glaciers range from 10 to 50 years (*e.g.*, Paterson, 1994; Oerlemans, 2001).

Glacier AX010 has been subjected to increasing temperatures over the past several decades, a trend that is expected to continue throughout this century (Christensen *et al.*, 2007). This represents roughly two e-folding times for Glacier AX010, long enough for the glacier to experience a near-complete response (by 2100) to climate conditions experienced by mid-century. The local climate is already untenable due to the lack of a persistent accumulation zone (WGMS, 2005), which is crucial for the survival of glacier (Pelto, 2010). It is therefore likely that Glacier AX010, in spite of responding relatively slowly to changing climate, may not survive thorough to the end of this century unless a cold and wet climate arrives. This is consistent with conclusion made in Adhikari and Huybrechts (2009).

On a final note, we recommend a few feasible methods that yield suitable estimates of response times for valley glaciers. Despite the simple underlying physics, the Jóhannesson model is recommended. It requires limited amounts of field data which are often available, but still yields reasonable estimates. If sufficient field data are available both in geometric and climatic details, the Harrison model should be used. It is an improvement to the Jóhannesson model that explicitly accounts for the important feedback between surface elevation and mass balance. Whenever available, dynamical ice-flow models can be used to estimate the response time by imposing a suitable step change in climate.

Acknowledgments

We thank the Vlaamse Interuniversitaire Raad (VLIR) for partially supporting this research when S. A. was a student at the Vrije Universiteit Brussel, Belgium. Thanks are also due to the Western Canadian Cryospheric Network (WC^{2}N), funded by the Canadian Foundation for Climate and Atmospheric Sciences (CFCAS), for hosting S.A. as a doctoral scholar at the University of Calgary, which made the completion of this research possible. Constructive comments from reviewers M. Pelto and S. Sugiyama and the editor A. Sakai helped us to improve the manuscript.

References

Adhikari, S. and Huybrechts, P. (2009): Numerical modelling of historical front variations and the 21st century evolution of Glacier AX010, Nepal Himalaya, Ann. Glaciol., **50** (52), 27–34.

Ageta, Y., Ohata, T., Tanaka, Y., Ikegami, K. and Higuchi, K. (1980): Mass balance of glacier AX010 in Shorong Himal, Nepal during the summer monsoon season, Seppyo, **41**, 34–41.

Bahr, D.B., Pfeffer, W.T., Sassolas, C. and Meier, M.F. (1998): Response time of glacier as a function of size and mass balance: (1) Theory, J. Geophys. Res., **103**, B5, 9777–9782.

Calmanti, S., Motta, L., Turco, M. and Provenzale, A. (2007): Impact of climate variability on Alpine glaciers in northwestern Italy, Int. J. Climatol., **27**, 2041–2053.

Christensen, J.H. and 16 others, (2007): Regional Climate Projections, In Climate change 2007: the physical science basis, Contribution of Working Group I to the Fourth Assessment Report of the Intergovernmental Panel on Climate Change (Solomon, S. and 7 others, eds.), Cambridge University Press, Cambridge, etc., 847–940.

De Smedt, B. and Pattyn, F. (2003): Numerical modelling of historical front variations and dynamic response of Sofiyskiy Glacier, Altai Mountains, Russia, Ann. Glaciol., **37**, 143–149.

Elsberg, D.H., Harrison, W.D., Echelmeyer, K.A. and Krimmel, R.M. (2001): Quantifying the effects of climate and surface change on glacier mass balance, J. Glaciol., **47** (159), 649–658.

Fujita, K., Kadota, T., Rana, B., Kayastha, R.B. and Ageta, Y. (2001): Shrinkage of Glacier AX010 in Shorong region, Nepal Himalayas in the 1990s, Bull. Glaciol. Res., **18**, 51–54.

Harper, J.T. and Humphrey, N.F. (2003): High altitude Himalayan climate inferred from glacial ice flux, Geophys. Res. Lett., **30** (14), 1764.

Harrison, W.D., Elsberg, D.H., Echelmeyer, K.A. and Krimmel, R.M. (2001): On the characterization of glacier response by a single time-scale, J. Glaciol, **47** (159), 659–664.

Huybrechts, P., De Nooze, P., and Decleir, H. (1989): Numerical modelling of Glacier d'Argentière and its historic front variations, In Glacier fluctuations and climatic change (Oerlemans, J., ed.), Kluwer Academic Publishers, 373–389.

Ikegami, K. and Ageta, Y. (1991): Ice flow of Glacier AX010 in the Nepal Himalaya, Bull. Glacier. Res., **9**, 17–22.

Jóhannesson, T., Raymond, C.F. and Waddington, E.D. (1989 a): A simple method for determining the response time of glaciers, In Glacier fluctuations and climatic change (Oerlemans, J., ed.), Kluwer Academic Publishers, 407–417.

Jóhannesson, T., Raymond, C.F. and Waddington, E.D. (1989 b): Time-scale for adjustment of glaciers to changes in mass balance, J. Glaciol., **35** (121), 355–369.

Kadota, T. and Ageta, Y. (1992): On the relation between climate and retreat of Glacier AX010 in the Nepal Himalaya from 1978 to 1989, Bull. Glacier Res., **10**, 1–10.

Kadota, T., Seko, K. and Ageta, Y. (1993): Shrinkage of Glacier AX010 since 1978, Shorong Himal, East Nepal, IAHS Publication 218, Snow and Glacier Hydrology, 145–154.

Kadota, T., Fujita, K., Seko, K., Kayastha, R.B. and Ageta, Y. (1997): Monitoring and prediction of shrinkage of a small glacier in the Nepal Himalaya, Ann. Glaciol., **24**, 90–94.

Kattelmann, R. (2003): Glacial lake outburst floods in the Nepal Himalaya: a manageable hazard? Nat. Hazards, **28**, 145–154.

Kayastha, R.B. and Harrison, S.P. (2008): Changes of the equilibrium-line altitude since the Little Ice Age in the Nepalese Himalaya, Ann. Glaciol., **48**, 93–99.

Klok, E.J. and Oerlemans, J. (2003): Deriving historical equilibrium-line altitudes from a glacier length record by linear inverse modelling, The Holocene, **13** (3), 343–351.

Leysinger Vieli, G.J.M.C. and Gudmundsson, G.H. (2004): On estimating length fluctuations of glaciers caused by changes in climatic forcing, J. Geophys. Res., **109**, F01007.

McClung, D.M. and Armstrong, R.L. (1993): Temperate glacier time response from field data, J. Glaciol., **39** (132), 323–326.

Naito, N., Ageta, Y., Nakawo, M., Waddington, E.D., Raymond, C.F. and Conway, H. (2001): Response characteristics of a summer-accumulation type glacier to climate changes indicated with a glacier fluctuation model, Bull. Glaciol. Res., **18**, 1–8.

Nye, J.F. (1952): A method of calculating the thickness of ice-sheets, Nature, **169** (4300), 529–530.

Nye, J.F. (1960): The response of glaciers and ice-sheets to seasonal and climate change, Proc. R. Soc. Lon. Ser-A, **256** (1287), 559–584.

Oerlemans, J. (1997): Climate sensitivity of Franz Josef Glacier, New Zealand, as revealed by numerical modelling, Arctic Alpine Res., **29** (2), 233–239.

Oerlemans, J. (2001): Glaciers and Climate Change, Swets and Zeitlinger BV, Lisse, p. 148.

Oerlemans, J. (2007): Estimating response times of Vadret da Morteratsch, Vadret da Palue, Briksdalsbreen and Nigardsbreen from their length records, J. Glaciol., **53** (182), 257–362.

Ohata, T. and Higuchi, K. (1980): Heat balance study on gla cier AX010 in Shorong Himal, east Nepal, Seppyo, **41**, 42–47.

Paterson, W.S.B. (1994): The Physics of Glaciers, 3rd ed., Pergamon, p. 496.

Pelto, M.S. (2010): Forecasting temperate alpine glacier survival from accumulation zone observations, The Cryosphere, **4**, 67–75.

Pelto, M.S. and Hedlund, C. (2001): Terminus behaviour and response time of North Cascade glaciers, Washington, USA, J. Glaciol., **47** (158), 497–506.

Raper, S.C.B. and Braithwaite, R.J. (2009): Glacier volume response time and its links to climate and topography based on a conceptual model of glacier hypsometry, The Cryosphere, **3**, 183–194.

Richardson, S.D. and Reynolds, J.M. (2000): An overview of glacial hazards in Himalayas, Quatern Int, **65**/**66**, 31–47.

Schwitter, M.P. and Raymond, C.F. (1993): Changes in the longitudinal profiles of glaciers during advance and retreat, J. Glaciol., **39** (133), 582–590.

Shrestha, A.B., Wake, C.P., Mayewski, P.A. and Dibb, J.E. (1999): Maximum temperature trends in the Himalaya and its vicinity: an analysis based on temperature records from Nepal for the period 1971–94, J. Climate, **12**, 2775–2784.

Shrestha A.B., Wake, C.P., Dibb, J.E. and Mayewski, P.A. (2000): Precipitation fluctuations in the Nepal Himalaya and its vicinity and relationship with some large scale climatological parameters, Int. J. Climatol., **20**, 317–327.

World Glacier Monitoring Service (WGMS), (1998): Fluctuations of glaciers 1990–1995, VII, Haeberli, W., Hoelzle, M., Suter, S. and Frauenfelder, R. (eds.), IAHS/UNEP/UNESCO.

World Glacier Monitoring Service (WGMS), (2005): Fluctuations of glaciers 1995–2000, VIII, Haeberli, W., Zemp, M., Fraunenfelder, R., Hoelzle, M. and Kääb, A. (eds.), IUGG/UNEP/UNESCO.