

Error correcting codes: problem set 2

1. Consider the parity check code of length 8 over \mathbb{F}_3 . This means that we add a ‘trit’ (this is the ternary version of a ‘bit’) to each message block of length 7 such that the sum of all 8 trits becomes 0.
 - Show that this code can detect all single errors and some double errors.
 - Is this code linear?
 - Is it a cyclic code? Justify your answer.
2. Consider a ternary code of length 6 that adds 3 check trits xyz to any message block abc such that $a + b = x$, $a + c = y$ and $b + c = z$.
 - How many errors can such a code correct?
 - Give generator and parity check matrices for this code.
3. We want to send binary messages of length 16. How many bits should be added to produce a *linear* single error correcting code? Can you do better by abandoning linearity?
4. Give a generator matrix for a code like in the previous problem.
5. Show that if a single error correcting linear code has a standard form generator matrix with all columns of even weight, then the code can detect double errors.
6. Show that in a binary linear code either all or exactly half of the codewords have even weight.
7. Let C and D be linear codes of the same length n and respective dimensions m and l . Suppose C has minimum distance d and D has minimum distance $2d$. Now construct a new code X whose codewords are $(u \mid u + v)$ with $u \in C$ and $v \in D$. Show that X is a linear code of length $2n$, of dimension $m + l$ and with minimum distance $2d$.
8. Enumerate all cyclic binary codes of length 5.