Error correcting codes: problem set 0

1. Suppose the binary repetition code of length 5 is used for a binary symmetric channel which has symbol error probability rate p. Show that the word error probability of the code is $10p^3 - 15p^4 + 6p^5$.
2. Show that a code with minimum distance 4 can be used simultaneously to correct single errors and detect double errors.
3. (a) Show that a $(3M, 2)_3$-code must have $M \leq 9$.
   (b) Show that a $(3, 9, 2)_3$-code does exist.
   (c) Can you generalize this to any alphabet with $q \geq 2$ letters?
4. Do there exist binary (n, M, d)-codes for the following parameter sets : $(6, 2, 6)$, $(3, 8, 1)$, $(4, 8, 2)$, $(5, 3, 4)$ and $(8, 30, 3)$? When not possible, explain why.
5. Show that if there exists a binary (n, M, d)-code, there exists a binary $(n-1, M', d)$ with $M' \geq M/2$. Deduce that $A_2(n, d) \leq 2A_2(n-1, d)$.
6. Prove that $A_q(3, 2) = q^2$ for any integer $q \geq 2$. [HINT: exercise 3]
7. Show that the number of inequivalent binary codes of length n containing just two codewords is n.
8. Show that $A_2(8, 5) = 4$ and that, up to equivalence, there is just one $(8, 4, 5)_2$-code.
9. Show that a $(q + 1, M, 3)_q$-code satisfies $M \leq q^{q-1}$.

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