On When a Disjunction is Informative

Ambiguous Connectives and a Realist Commitment to Pluralism

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Abstract. In this paper we investigate the topic of ambiguous connectives, as it was recently explored by Paoli, from an informational perspective. That is, starting from the framework of informational pluralism (Allo, in press, submitted), we ask what it means for a message of the form 'A `or` B', to be informative.
Using these disjunctive messages as an example, we answer three traditional objections to substructural logic and logical pluralism, and eventually show that the linear or relevant logician's road to unambiguous connectives is consistent with informational pluralism.

Keywords: semantic information, logical pluralism, substructural logics

1. Pluralism about Consequence and Content

Following a suggestion of Beall & Restall's, it is our aim in this paper to exploit logical pluralism as a means to recognise a distinct, but not unrelated kind of pluralism. To the latter, we refer as informational pluralism. We start by outlining two distinct arguments in favour of logical pluralism.

First, we have the argument based on an ambiguity pointed out by Beall & Restall in what they call the Generalised Tarski Thesis (GTT).

> (GTT) A conclusion $\mathcal{A}$ follows from premises Σ iff any *case* in which each premise in Σ is true is also a *case* in which $\mathcal{A}$ is true. (2006, 29)

Their argument for logical pluralism specifically rests on an ambiguity in the use of the notion of a *case* within any fairly standard description of logical consequence as *truth-preservation*. Avoiding the traditional, but not always explicitly named restriction to complete and consistent cases, the GTT lays the ground for more than one logical system.

A second argument is a modification of an early objection to relevant logic due to Hanson (1980), and relies on an alternative account of logical consequence as *content-nonexpansion*.

> (CN) An argument is valid just in case the content of the conclusion does not exceed the combined contents of the premises.

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Whereas the first argument clearly and intendedly aims at a pluralist conclusion, the second one was originally presented with a monist interpretation in mind. Showing that a coherent account of content could be given such that *CN* exactly yields the classical notion of consequence, Hanson wishes to defuse the traditional relevantist's claim that logical consequence as truth-preservation ought to be extended with content-preservation. As a consequence, his provision of a classically minded notion of content suffices to show that if consequence needs both truth- and content-preservation, then classical logic is just fine and the mainstream relevantist incentive to revise *Logic* readily vanishes.

The first claim to be defended in this paper, is that Hanson's argument only works as an objection to, on the one hand, the relevantist monist, and, on the other hand, those relevant logicians who adhere to Anderson & Belnap's traditional (and admittedly rather vague) account of relevance. It will thus be argued that *CN* provides an equally compelling reason to be a logical pluralist as the *GTT* does in Beall & Restall's exposition. To do so we adopt *content* and *content-containment* as elementary notions, and argue for the thesis of informational pluralism using a suitably generalised version of the *Inverse Relationship Principle* (see e.g. Barwise, 1997).[1]

> (GIRP) The informational content of a piece of information is given by the set of cases it excludes.

Jointly, *GIRP* and *GTT* provide all we need to sketch the analogy between Beall & Restall's original argument, and the one we wish to defend here. An early version of informational pluralism was introduced in Allo (in press) and further elaborated in Allo (submitted). Continuing along the lines set out in those papers, we specifically focus on issues related to the ambiguity of the classical disjunction, and thereby revisit some famous controversies that have been around since the systematic development of relevant logic by Anderson & Belnap. These are, to name but a few, the notion of relevance itself, the alleged opacity of the Routley-Meyer semantics, and the distinction between so-called intensional and extensional connectives.

The next two sections disambiguate the notion of content and rehearse the basics of its pluralist interpretation (Allo, submitted). In section 4 an informal model that interprets the formal semantics for a weak relevant logic (see apppendix) is given. Finally, this model is used to evaluate some objections against substructural logics and their pluralist interpretation.

2. Situated and Worldly Content

Adopting a strategy similar to Beall & Restall's, a pluralist individuation of the informational content of a message is obtained by varying among suitable precisifications of the notion of a *case* within the *GIRP*. Rather straightforwardly, this yields an account of classical, relevant, or intuitionistic content, depending on whether we let cases be possible worlds, situations, or constructions. Bypassing the underlying logical systems involved, the corresponding accounts of content can be derived directly on the basis of the modal space used in the respective frame-semantics. For reasons of focus, we choose to leave the intuitionist option aside, and consider its classically and relevantly inspired interpretations only.

Whilst the classical approach—originally elaborated by Bar-Hillel & Carnap (1952) in terms of state descriptions—quite expectedly identifies the content of A with the set of possible worlds which do not support A, a mere enlargement of the modal space suffices to obtain a relevantly inspired account of content. That is, using Routley-Meyer frames, we no longer evaluate formulae at possible worlds, but at logical points which should neither be consistent nor complete (cfr. appendix). Following standard usage we call such points situations; the consistent ones are referred to as *possible*, the inconsistent ones as *impossible*. We should furthermore remark that possible worlds represent a peculiar kind of situations, namely the complete and consistent ones. Even more than it is the case for Beall & Restall's logical pluralism, this last consideration is constitutive for our informational pluralism.

Remark first that, using the frame-semantics outlined in the appendix, one succeeds in identifying possible worlds as a well-defined class of situations.

$$s \Vdash A \text{ or } s \Vdash\!\sim A \text{ iff } s = s^*$$
$$\{s : s = s^*\} \subset S$$

This enables us to avoid any further reference to possible worlds as a primitive notion. Thus applying the intuitive characterisation of content expressed in the *GIRP* with respect to this enlarged set of logical points, we obtain two distinct content-individuations:

$$CONT_s(A) = S \setminus \{s : s \Vdash A\} \quad (1)$$
$$CONT_w(A) = \{s : s = s^*\} \setminus \{s : s = s^* \ \& \ s \Vdash A\} \quad (2)$$

In analogy with Mares' (2004) interpretation of relevant implication as situated inference, we propose to call (1) the individuation of the situated content of a message A. Likewise, we call (2) the worldly content of A. Showing the value of these two distinct notions, we need to argue

that (a) they genuinely disagree on the content of some messages, and (b) they both correspond to something real.

Since both issues are extensively discussed in Allo (submitted), we only briefly recapitulate the argument by means of an example using a message of the form 'A `or not-`A'. Formally, the individuations given above give us little to doubt about. $CONT_w(A \texttt{ or not-} A) = \emptyset$, whereas $CONT_s(A \texttt{ or not-} A)$ encompasses the non-empty set of all situations which do not decide A.[2] Despite having obtained a genuine disagreement using both formal characterisations of content, we still need to explain what it means for one and the same message to be assigned a non-empty situated content, but at the same time also have an empty worldly content. Put simply, we hold that each evaluation refers to a distinct feature: [W] the sender of such a message could have been in any possible world (the empty worldly content), [S] but meanwhile she could not have been in any situation in any such world (the non-empty situated content). Only an A-deciding situation could have provided the required (factual or explicit) evidence for a truthful assertion, a requirement trivially satisfied by possible worlds or worldly situations, but not by arbitrary situations.

Crucially, one should keep in mind that the existence of a worldly perspective is a peculiar issue. Not only can it fail to exist w.r.t. an arbitrary situation (i.e. $s \sqsubseteq s^*$ does not hold for all $s \in S$), but its existence is not a persistent property either (i.e. $s_1 \sqsubseteq s_1^*$ & $s_1 \sqsubseteq s_2 \not\Rightarrow s_2 \sqsubseteq s_2^*$). This very feature is customarily referred to as the existence of so-called impossible situations, situations that are formally inconsistent and hence have no refinement that is a possible world. Without discussion, this is the single most controversial aspect of the modal space we use. Yet, we shall not dwell upon this issue, but just mention a few points. Firstly, on the formal level impossible situations do not pose any problem (see the appendix). Since their advent, many interpretations of impossible situations have been given, but none of them shall ultimately concern us here (e.g. the essays in Priest (1997), or more recently Sequoiah-Grayson (2006)). The only explanation of impossible situations we need to endorse is fairly minimal. It solely depends on the fact that no information A that is supported by a situation s can convey the information that s has no impossible refinements. As a special case of this, we equally have that no A supported by s actually conveys the information that s itself is possible. In short, even though inconsistent situations are plainly impossible, no message can by itself convey the information a receiver requires to conclude that the sender of that message does not find herself in an inconsistent situation. Though impossible situations can be ruled out as a matter of logic alone, no message can explicitly express the need to do so.

3. Factual and Constraining Content

With the standard relevantist recapture of the disjunctive syllogism in mind, the analysis provided in the previous section could be questioned on several levels. Before answering such objections, it is better to rehearse the basic aspects of the controversy surrounding the classical inference-rule of disjunctive syllogism. Several doubts regarding the status of DS probably find their origin in Lewis' independent argument for *Ex Falso Quodlibet*, an argument-form rightly considered the basis of the most problematic paradox of material and strict implication.

$$\frac{\dfrac{A}{A \texttt{ or } B}\,(\texttt{or } I) \qquad \sim A}{B}\,(DS^{\texttt{or}})$$

Rejecting this argument in view of the lack of *relevance* of the premises for the conclusion, the mainstream relevantist answer to *disjunctive syllogism* has consisted in the rejection of the classical rule of $DS^{\texttt{or}}$. However, when confronted with the need to explain the general usefulness and (apparent) correctness of the very same inference-rule, relevant logicians generally provide a more fine-grained account of its rejection (Read, 1981).[3] Namely, the invalid rule is to infer B from $A \sqcup B$ and $\sim A$, whereas its valid version is formalised in the rule $(DS^{\oplus})$ given below. To the contrary the classical rule of *Addition* ($\texttt{or } I$) cannot be used to obtain $A \oplus B$ from either A or B, but is only acceptable as $(\sqcup I)$.

$$\frac{A \oplus B, \sim A}{A}\,(DS^{\oplus}) \qquad \frac{A}{A \sqcup B}\,(\sqcup I) \qquad \frac{B}{A \sqcup B}\,(\sqcup I)$$

Put this way, it is not hard to see why to the relevantist the classical *Ex Falso* rests on a spurious equivocation contained within the classical disjunction. Following the terminology used in Paoli (2005), we call $\oplus$, and its dual $\otimes$ the group-theoretical disjunction and conjunction, whereas $\sqcup$ and $\sqcap$ are designated as lattice connectives. To avoid confusion, we stick to '`or`' and '`and`' when referring to ambiguous connectives.

The problem this way out poses for a pluralist interpretation of informational content is threefold. Namely, (a) if correct, it renders the pluralist account of the content of a message 'A `or not-`A' incomplete, (b) it threatens the need for classical logic and hence also the need for a notion of worldly content, and (c) if only an ad hoc solution, the problem of explaining away the actual uses of DS (when there is no worldly perspective) remains. For starters, we do acknowledge that in order to provide an exhaustive account of informational content, the

notions of situated and worldly content given above do not suffice as an explanation of the content of a message of the form 'A `or` B'. They need to be extended with a content-individuation that incorporates the frame-interpretation of $A \oplus B$. This provides an answer for the concern expressed under (a); solutions for the objections mentioned under (b) and (c) follow in section 5.

In order to understand the content conveyed by (a message of the form) $A \oplus B$, two distinct perspectives can be adopted. First, reading of the relevant satisfaction-clauses we notice that while $s \Vdash A \sqcup B$ conveys itself information about the truth of A and B at s, $s \Vdash A \oplus B$ remains silent on these matters. This suggests that an all too simple interpretation of the *GIRP* with respect to messages correctly disambiguated using group-theoretical connectives might very well be fallacious. Indeed, it is an often made remark that the connective $\oplus$ primarily possesses inferential force. Not only does it satisfy a version of *DS* (which accounts for its inferential force), but the failure of addition points to the other side of this issue, namely its lack of factual content.

The solution we propose, is to treat the content of $\sim A \oplus B$ as the conditional content of B, given A. A move warranted by (i) the equivalence of $A \rightarrow B$ and $\sim A \oplus B$, and (ii) the assumption that identifying the content of a conditional with the conditional content of the consequent given the antecedent is the right attitude to capture the inferential content of a message using $\oplus$ as its main connective.

Implementing this insight, we first need to recall that traditionally (worldly) conditional content is defined in terms of content simpliciter, see (3). As a corrolary, we also note that when applied to worldly content this way of defining conditional content does not discriminate between the factual and the inferential content of a message. The collapse given in (4) is inherent to the notion of worldly content, and plainly mimics the definition of material implication as $\sim A \vee B$.

$$CONT_w(B \mid A) = CONT_w(A \,\&\, B) \setminus CONT_w(A) \tag{3}$$

$$CONT_w(B \mid A) = CONT_w(\sim A \vee B) \tag{4}$$

However a similar equivalence between the content of B on the condition that A, and the content of $\sim A \oplus B$ is clearly intended, one must be cautious not to interpret the antecedent or condition as conveying factual content itself. In order to avoid this confusion, we first define the set of c-accessible A-situations (see Mares, 2006) that shall serve as an individuation of the content conveyed by the antecedent as:

$$\begin{aligned} A^{\sqsubseteq}_c &:= \{s_i : \exists s(s \Vdash A \,\&\, s \sqsubseteq_c s_i)\} \\ &:= \{s_i : \exists s(s \Vdash A \,\&\, \forall B(c \Vdash B \Rightarrow s_i \Vdash A \otimes B))\} \end{aligned} \tag{5}$$

This can subsequently serve to give a content-individuation of B given A that does not collapse into the set of all situations that support A and

$\sim B$. In other words, by giving an intentional or group-theoretical reformulation of (3), it bypasses the factual interpretation of conditional content, and gives it a purely constraining interpretation instead.

$$\begin{aligned} \mathrm{CONT}_c(B \mid A) &= A_c^{\sqsubseteq} \setminus (A_c^{\sqsubseteq} \cap \{s : s \Vdash B\}) \\ &= A_c^{\sqsubseteq} \setminus \{s : s \Vdash A \otimes B\} \end{aligned} \tag{6}$$

Informally, the situated constraining content of B, given A is best understood as the proportion of c-accessible (or informationally linked) A situations that do not eventually support B. Importantly, it can be shown that (6) individuates a proposition.

This leaves us with three distinct accounts of content: [W] standard worldly content, [S] situated factual content, and [C] inferential or situated constraining content. A few features of each of these need to highlighted. In the first place we must conclude that worldly content accounts for both the inferential or constraining as well as for the factual content of a message. Secondly, it can be noted that the strict distinction between situated factual content and situated constraining content is perfectly mirrored by the distinction between the lattice and the group-theoretical connectives. Finally, the fact that classical logic cannot discriminate between these two classes of connectives provides a sufficient ground to claim that from a worldly perspective the constraining content of a message *just is* its factual content. It is crucial to our pluralist enterprise that this latter fact is not dismissed as a spurious equivocation, but rather treated as a natural restriction on the discriminatory power that is inherent to the worldly perspective on content.

Before providing a model in which each of these notions receives a plausible interpretation, it can already be noted that all three [W], [S], and [C] provide a specific instance of a general insight into the nature of content and informativeness. Namely, the *dual to logic* intuition:

> (DTL) X is informative only to the extent that X lies outside the scope of logical consequence alone.

It can thus easily be checked that [W] is dual to classical logic, [S] to relevant tautologies, and [C], provided it is evaluated with respect to all $c \in \mathit{Log}$ only, is dual to relevant consequence.

4. Modelling Content

A simple model that allows one to interpret a formal model of informational content involves at least: (i) a sender, (ii) a receiver, and (iii) an event observed by the sender. What we evaluate in such a

model is in the first place the content a message (truthfully) conveys to the receiver about an event the sender observes. As before (Allo, submitted), we choose a simple game as the event reported upon. A standard game-tree is used to establish what counts as the sender's evidential situation.

A game of *tic-tac-toe* is used as an actual example, and formalised in the following manner. Atomic formulae referring to a move in the game are of the form $P_i a_j$ where P_i is a position on a 3×3-board, and a_j refers to the turn in the game (such that a_1 is the first move, etc.).

$$\begin{aligned} \mathsf{Position} &= \{P_1, P_2, \ldots, P_9\} \\ \mathsf{Turn} &= \{a_1, a_2, \ldots a_9\} \\ \mathsf{Atom} &= \{P_i a_j \mid P_i \in \mathsf{Pos} \;\&\; a_j \in \mathsf{Turn}\} \end{aligned}$$

A basic description of an evidential situation s_i or sequence of board-configurations is a finite conjunction of atomic formulae. For instance, $s_3 \Vdash P_1 a_1 \sqcap P_5 a_2 \sqcap P_7 a_3$ describes the board resulting from subsequent additions of a cross on position 1, a nought on 5, and again a cross on 7. Given the clearcut connection between descriptions of this kind and the evidential situation that supports them, a formulation using lattice-connectives is obviously warranted.

Since every move in the game can also be seen as (in principle) precluding some other moves, we introduce extended descriptions of an evidential situation. This is obtained by enhancing the basic description with formulae obtained through the application of the following two rules:

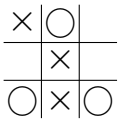
$$s_i \Vdash P_i a_j \;\&\; P_k \in (\mathsf{Position} \setminus \{P_i\}) \;\Rightarrow\; s_i \Vdash\!\sim P_k a_j \tag{7}$$

$$s_i \Vdash P_i a_j \;\&\; a_l \in (\mathsf{Turn} \setminus \{a_j\}) \;\Rightarrow\; s_i \Vdash\!\sim P_i a_l \tag{8}$$

As we seek to explain a modal space wherein $s \sqsubseteq s^*$ does not hold for arbitrary s, the set of all board-configurations cannot be restricted to those that respect the rules. Specifically, the set S includes several impossible configurations (most evidently boards containing overlapping noughts and crosses), while the subset $\{s : s \sqsubseteq s^*\}$ can be said to contain all and only those boards which result from possible games. By considering the rules given above as a means to encode some non-overlap rules within the descriptions themselves, impossible configurations are just those configurations which support an inconsistent description; impossibility and negation-inconsistency are made to coincide. The incorporation of such impossibilities is motivated by the paraconsistently inspired concern that within this example, a non-reglementary game should not be considered a trivial game (a game wherein each move is actually made), but only an impossible game (a game that no move can turn into a completed game). Alternatively, the

set S can be thought of as the set of all boards someone who observes, but does not know the rules of the game can conceive of. Its subset $\{s : s \sqsubseteq s^*\}$, however, is still constrained by what can actually occur as a node in a game-tree.

So far, this only suffices to model evidential situations one can describe using lattice-connectives and negation only. A further extension for the group-connectives remains to be given. Consider the following board-configuration and matching description:



$$s \Vdash P_1a_1 \sqcap P_7a_2 \sqcap P_5a_3 \sqcap P_9a_4 \sqcap P_8a_5 \sqcap P_2a_6$$

Figure 1.

Informally, we certainly would like to say that given this board, a sender who knows the game should be able to communicate more than what is true at s only (i.e. the closure of the description under $\sqcup$, $\sqcap$, and the two additional rules given above). For instance, it makes sense that $P_3a_7 \oplus P_4a_7 \oplus P_6a_7$, or even that $P_6a_7 \rightarrow (P_3a_8 \oplus P_4a_8)$. Hence, one could suggest to supplement the sender's evidential situation by means of a game-tree depicting all constraints on the possible extensions of the actual board-configuration.

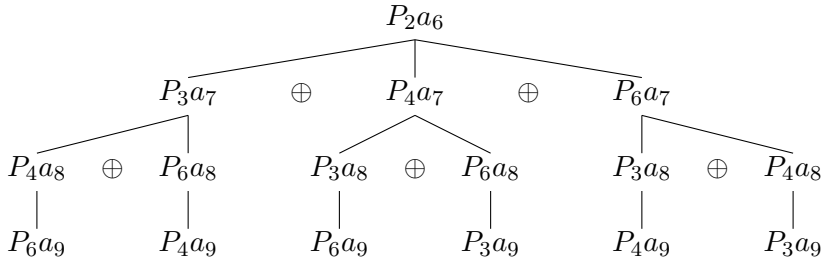


Figure 2.

In the light of previous game-theoretical treatments of linear logic, it is an obvious move to explain the meaning of the group-disjunction $\oplus$ on the basis of choices a player can make in a game. The novelty of the present approach is, therefore, situated on a different level; viz. the use of the *method of abstraction* (Floridi & Sanders, 2004) as a formal approach to the receiver's failure to discriminate between lattice and group-theoretical disjunctions—that is, what Humberstone (2005) calls the logical discrimination.

Concretely, the modelling involves the introduction of two perspectives or levels of abstraction, depending only on the ability to discriminate between messages sent by a group of agents, and messages sent by a single agent. Additionally, we stipulate that the receiver's perspective is such that any message is perceived *as if* it originated from a single agent. The receiver, in other words, is modelled with respect to the lesser discriminating level of abstraction. We start, however, with the description of the first, more elaborated level.
Let, as said, the sender be a group of agents such that:

> [FACT] A first agent disposes of an extended evidential situation and can only send messages insofar as they are supported by this evidence. Informally this corresponds to the actual game-board, and the application of (7) and (8). Evidently, every message supported by this board can be expressed using lattice connectives and negation only.
>
> [CONS] A second group of agents disposes of constraining information regarding the possible ways in which the actual board-configuration can be extended. This group of agents can only send messages through collaboration. For instance, if n such agents cover all possible moves $P_i a_{10-n}$ given the actual board, they are entitled to convey the message $P_i a_{10-n} \oplus \ldots \oplus P_j a_{10-n}$ where each disjunct is such a move.

Referring to the game-tree depicted above, n agents can send a message $P_i a_{10-n} \oplus \ldots \oplus P_j a_{10-n}$ iff any disjunct of the message labels an immediate successor of the root, and every label of an immediate successor of the root is a disjunct of the message. More generally, $n+1$ agents can send a message $P_i a_{9-n} \rightarrow (P_j a_{10-n} \oplus \ldots \oplus P_k a10-n)$ iff any disjunct labels an immediate successor of a node labelled with $P_i a_{9-b}$, and every label of an immediate successor of a node labelled with $P_i a_{9-b}$ is a disjunct.

First Example: $P_3 a_7 \sqcup {\sim} P_3 a_7$ and $P_3 a_7 \oplus {\sim} P_3 a_7$
Evaluating these messages, a disagreement between the notion of situated factual content and situated constraining content surfaces. Whereas the lattice-formulation can only be truly asserted on the condition that the seventh move has a definite value, the group-formulation is not bound by this requirement. Hence, the former does convey factual content, while the latter (if evaluated with respect to *Log* only) does not put any real constraint upon the possible evolutions of the game,[4] it is informationally empty.

Since we wish to understand genuine constraining content as extra-logical content, it is a normal and harmless assumption to evaluate constraining content with regard to logical constraints only. A better appreciation of this approach can be obtained in a second example.

Second Example: $P_6a_7 \rightarrow (P_3a_8 \oplus {\sim}P_3a_8)$
Individuating the (constraining) content conveyed by this message, we use $CONT_c(P_3a_8 \oplus {\sim}P_3a_8 \mid P_6a_7)$.

$$\begin{aligned} CONT_c(P_3a_8 \oplus {\sim}P_3a_8 \mid P_6a_7) &= CONT_c(P_3a_8 \rightarrow P_3a_8 \mid P_6a_7) \\ &= CONT_c(P_3a_8 \mid P_6a_7; P_3a_8) \\ &= (P_3a_8 \otimes P_6a_7)^{\sqsubseteq}_{c_1;c_2} \setminus \{s : s \Vdash P_3a_8 \otimes P_3a_8 \otimes P_6a_7\} \end{aligned}$$

Keeping in mind that $B \rightarrow (A \rightarrow A)$ should not be a theorem of relevant logic, it is to be expected that the constraining content of the message above is non-empty. Yet, since in the previous example we settled that a message of the form $P_ia_j \oplus {\sim}P_ia_j$ had to be informationally empty, the failure of that analysis for this new message needs to be made explicit. Specifically, the constraint against which the evaluation of the whole message is carried out needs to be brought to the surface. In general, theoremhood is judged at a logical situation or constraint. This is why, if we take *DTL* seriously, constraining content is to be judged against a logical constraint too. It is, however, a mistake to infer from this insight that *any* appeal to a constraint is therefore an appeal to a logical constraint. For instance, the judgement that $B \rightarrow (A \rightarrow A)$ is not relevantly valid explicitly bears upon the possibility of evaluating the consequent $A \rightarrow A$ with respect to all constraints. This very feature, is recovered within the content-individuation described above by means of $(P_3a_8 \otimes P_6a_7)^{\sqsubseteq}_{c_1;c_2}$,[5] a proposition that possibly exceeds *Log*. Comparing thus the following two individuations, the distinction can be explicitized.

$$CONT_c(A \mid A) = A^{\sqsubseteq}_c \setminus \{s : s \Vdash A \otimes A\} \quad (9)$$

$$CONT_c(A \oplus {\sim}A \mid B) = (A \otimes B)^{\sqsubseteq}_{c_1;c_2} \setminus \{s : s \Vdash A \otimes A \otimes B\} \quad (10)$$

Indeed, whereas (9) individuates the proportion of c-accessible A-situations that do not support A too for the basic case where $c \in Log$ only, (10) individuates the proportion of $c_1;c_2$-accessible $A \oplus B$-situations that do not support A too on the weaker assumption that $c_1 \in Log$. No matter how much one knows about c_1, the stronger assumption that $c_1;c_2 \in Log$ cannot be derived from the former, and consequently one cannot in general rule out the possibility of (10) being judged against an impossible constraint.[6] Even then, it seems that the weak assumption that $c_1 \in Log$ is all one needs to comply with *DTL*. While the former argument only settles the case on the formal level, the connection with the game interpretation is quickly made. Assume, for instance, that the receiver cannot rule out that the actual board-configuration, say s, is impossible. In such a case, even though it is natural for this receiver to judge the constraining content of a message with respect to a $c_1 \in Log$ it does not have the resources to rule out that the result of applying c_1 to s (i.e. any x such that $s \sqsubseteq_{c_1} x$), is

possible too. But this is exactly what is required to settle that the composed channel $c_1; c_2$ is logical too (cfr. appendix).

Trying, in turn, to formalise the less discriminating level of abstraction associated with the receiver's perspective, it is the distinction between $\sqcup$ and $\oplus$ which ought to disappear. The classical connectives, however, cannot serve that purpose since, (i) using the classical disjunction as a means to render a message for which $n > 1$ agents collaborated would turn constraining content into factual content, and (ii) using the classical disjunction to render a single-agent disjunction would erroneously assign some inferential strength to a purely factual message. To the receiver, the group of agents is just seen as a simulation of a single agent. Consequently, the incoming messages are, even though the result of a distributed system involving both single and collaborating agents, considered as a standard set of premises wherein the fine-grained structure derived from $(S, Log, \sqsubseteq_{\_}, *)$ is lost. Unfortunately, in such a case the content of a message cannot safely be individuated by simulating the receiver's perspective through a set of possible worlds only. There simply is no warrant for assuming the existence of a non-empty set of possible worlds or *worldly perspective*. In other words, the previously made remark that a worldly perspective need not exist remains, even after the introduction of a notion of constraining content.

The level of abstraction that duly incorporates the ability of discriminating messages sent by one or more agents is best rendered using both the lattice and group-connectives. Informational content is, with respect to that level of abstraction, correctly rendered using two unambiguous notions of content. Namely, $CONT_c$ for the cases described under [CONS] and $CONT_s$ for the cases described under [FACT]. Coming to the less fine-grained receiver's perspective, the logical discrimination assumed at the previous level is totally lost. Yet, the classical collapse of previously distinct connectives may not apply as a rendering of this reduced discrimination. All one can say, is that unbeknownst to the receiver content is effectively governed by $CONT_s$ and $CONT_c$. These considerations do show that disjunctive messages are sometimes rightly described by means of the ambiguous connective '`or`', while their content should not necessarily be governed by their classical properties. In that sense, the receiver's perspective we have modelled, implicitly recaptures what Paoli (2003) calls the ambiguity of our natural language *or*.

As such, the framework outlined in this section suffices to tackle three subsequent objections to substructural logic, logical pluralism, and more generally the individuations of informational content based on them.

5. Three Objections Revisited

Burgess' Objection

In his "Relevance: A Fallacy" Burgess fiercely argues against the distinction between $\sqcup$ and $\oplus$ proposed by some relevantists, and especially against their contention that each apparently valid instance of $DS^{\texttt{or}}$ could be recast as an application of $DS^{\oplus}$. Upholding that, if not simply ad hoc, the distinction between lattice and group-theoretical connectives could be shown to reduce to a mere subjectivization of relevance which rests on a confusion of implication with inference, Burgess is not prepared to accept this distinction as a logical distinction.

> Relevantism would reduce to the position that (IA) [$DS^{\texttt{or}}$] is valid when and only when one's grounds for asserting $p \vee q$ are something other than the simple knowledge that q. Such a position, however, looks suspiciously like a confusion of the criteria for the *validity* of a form of argument with the criteria for its *utility*, a confusion of logic with epistemology. (...)
> Thus if [Anderson & Belnap] intend by "relevance" something less than objective, they are highly remiss in failing to alert their readers to the fact; while if "relevance" is supposed to be impersonal, then the claim that the relevantistic position is (even in a weak sense) compatible with commonsense and accepted mathematical practice succumbs to the counterexamples presented above. (Burgess, 1981: 103)

Accepting Burgess' claims, one can only conclude that there is no viable notion of informational content beyond that of worldly content. The resulting position is that of a classical monist, a view that squarely contradicts the findings of the preceding sections.

What allows us to escape this conclusion, is the adoption of a broadly Dretskian attitude when reformulating this first objection in terms of content and content-containment. It is therefore hardly surprising that our response departs in many ways from earlier defences of relevantly inspired recaptures of the disjunctive syllogism (Mortensen, 1983, 1986; Read, 1981, 1983, 1988).

The crucial point of our argument is already implicit in the previously made assertion that the content of a message 'A `or` B' is, even in those cases where the receiver lacks the resources to correctly disambiguate it, either correctly individuated using $CONT_s$ or using $CONT_c$. In short: the content a message conveys is determinate, and this is so independently of the receiver's actual knowledge of that content. Clearly, such a remark is reminiscent of many points in Dretske's "Knowledge and The Flow of Information".

> The explanation of this "paradox" lies in the fact that the information (...) can be communicated over a channel without the receiver's knowing

> (or believing) that the channel is in a state such as to transmit this information. The receiver may be quite ignorant of the particular mechanisms responsible for the delivery of information–holding no beliefs, one way or the other, about the conditions on which the signal depends.
> Information (and therefore knowledge) about a source depends on a reliable system of communication between a source and receiver—not on whether it is known to be reliable. (Dretske, 1999: chapt. V)

Reconsidering Burgess' objection in this light (and not only with respect to the view he explicitly challenges, viz. Anderson & Belnap's), one can easily point to a first confusion on his side. To wit, if the distinction between $A \sqcup B$ and $A \oplus B$ were just a matter of subjectivity—of knowledge or the lack of knowledge of either A or B—then the ability to discriminate between them had better be assigned to the receiver. Yet, this is exactly what our model denies. The information explicitly available to the receiver is mediated by a level of abstraction whose main characteristic is its failure to discriminate between messages sent by one or by more agents. The distinction between lattice and group-theoretical connectives is therefore equally unavailable to the receiver. Whether the information conveyed by the ambiguous message 'A `or` B' is factual or constraining is judged independently from the receiver's knowledge of the truth of A or B. Thus, if the model presented in the previous section adequately reflects the distinction between lattice and group-theoretical connectives, this very distinction cannot be explained in terms of the subjectivity of the receiver. Yet, since Burgess mentions the grounds for asserting a disjunction instead of the resources to recognise it, we need more to meet his challenge.

More important therefore, is the fact that the distinction cannot be recast in terms of subjectivity of the sender either. Given an accurate use of the distinct levels of abstraction at work, it can be shown that a reduction to the information available to the sender is, if not straightforwardly based on a confusion, then at least based on an incomplete understanding of the context of communication. The tempting mistake rests on the intuitive truth of the following claim: a truthful message 'A `or` B' conveys the information associated with $A \sqcup B$ iff the sender could—given the evidence presently at its avail—have sent a more informative message that entails either A or B; if not, then it conveys the content associated with $A \oplus B$. Quite rightly, the ability of conveying the information associated with one or another disambiguation can be reduced to the ability to convey the information associated with its subformulae. Nevertheless it only provides an accurate description given the assumption that messages are perceived as coming from a single agent (as we have seen, the receiver's perspective). This assump-

tion, however, amounts to reducing our understanding of the context of communication to a single level of abstraction.

The view we wish to defend encompasses two claims. First, the content effectively conveyed by a message (alternatively, what counts as its correct disambiguation) is a determinate matter in that it is independent from the receiver's previously acquired information. This defuses Burgess' objection that the relevantist confuses inference with implication. Secondly, what content is effectively conveyed can only be recast in simple epistemological terms if the sender is perceived as a single agent, and—as discussed above—the latter depends on the adoption of the lesser discriminating level of abstraction. As soon as the 'sender' is recognised as a group of agents that possibly need to collaborate to send a message,[7] the epistemological explanation for the distinction is provided with a second, not purely epistemic ground. This, in the end, shows the non adhocness of the distinction.

A final objection to Burgess' criticism refers to his failure to recognise the possibility that information might not only be distributed over distinct agents, but might be so unbeknownst to the receiver of the message that conveys this information. Since, as argued in a previous section, the failure to discriminate the fine-grained structure of the information cannot safely be described using a worldly perspective only (i.e. classical logic), a reason not to be a classical monist is obtained.

Read's Objection

Being one of the most consistent advocates of the distinction between lattice and group-theoretical connectives, any objection to informational pluralism derived from Read's position is bound to be diametrically opposed to the one discussed above. Despite the vagueness of the label, we shall refer to his position as a *relevantist's monism*, and devote our attention to the consequences of his rejection of both classical logic and a classical meta-theory. As voiced on many occasions, it is Read's opinion that the Routley-Meyer semantics for relevant logic is—as Meyer himself claimed before—indeed a *gentile semantics* (1988). It uses an extensional language to reconstruct an essentially intensional consequence relation. More recently, in objecting to logical pluralism, the same diagnosis was advanced as a central part of his argument: "If Beall and Restall insist on doing semantics classically, then they are just classical logicians" who think of non-classical logics as incomplete instead of truly rivalling logics (2006: 207–9). Their pluralism depends more a failure to really reject classical logic, than on a robust position in itself.

On the face of it, the informational pluralism outlined above falls prey to both sides of Read's objection. Not only does the model presented in section 4 take the Routley-Meyer semantics very seriously, but its explicit endorsement of a notion of *worldly content* boils down to an overt acceptance of classical logic. Our defence against a relevantist's monism shall comprise two parts; a first part to defend our modelling, and a second one specifically in favour of the indispensability of a notion of *worldly content*.

Regarding the first, a fairly recent tradition of formulating intuitive interpretations for formal semantics previously deemed opaque, confers some initial plausibility to our enterprise. The fact that two such models (Mares, 1997; Restall, 1995) fall back on situation-semantics and information-flow furthermore suggests the viability of an information-based reading of the semantics. Yet, it speaks in our model's favour that by avoiding the explicit references to information-flow (according to some an equally obscure notion) both Mares and Restall make use of, it cannot as easily be dismissed either. After all, our initial aim was not as much to provide an intuitive interpretation of the semantics of relevant logic as well as an attempt to devise a more fine-grained account of content and content-containment.[8] In that perspective, the model based on communicating agents comes first, and—however it makes the distinction between lattice and group-theoretical connectives less obscure—its main purpose is not to defend an extensional semantics as a non-gentile semantics. Even though one might conclude that it makes the Routley-Meyer semantics more respectable, this conclusion is largely immaterial to the pluralist position about informational content defended here. The pluralism we take to be unavoidable follows from the interpretation in terms of communicating agents, not from the fact that an extensional semantics is used.

Since it touches upon the core of our pluralism, Read's rejection of classical logic is more troublesome. As he has it, relevant logic discriminates between two accounts of consequence, one material and one relevant. Classical logic, to the contrary, conflates them. So far, we cannot but agree: the first four sections of this paper recognise that from a worldly perspective constraining and factual situated content fully coincide. Yet, beyond this point the disagreement with Read's monism is blatant. To see how this affects our position, we need to go back to the intuition behind *DTL*: the fact that informational content and logical consequence should be considered as dual. If Read is right in rejecting both material and classical consequence, of the three measures we proposed only $CONT_c$ survives. As from a monist perspective they cannot comply with *DTL* too, the material $CONT_s$ and the classically minded $CONT_w$ only count as formally correct precisifications

of *GIRP*—mere artefacts of our usage of a *case*-based semantics. This leaves us with a number of distinct issues to resolve. Not only do we need to establish the usefulness of $CONT_s$ and $CONT_w$ (*qua* kinds of content), but a coherent case in favour of DTL remains to be presented for each of them too.

As an argument for the relevance of $CONT_s$, it should be sufficient to recall the previously suggested interpretation of situated factual content. Viz. A is non-empty only if A could not have been communicated by a sender in any situation. Recognising the partiality of information-states, and the distributed nature of information in a complex environment, the notion of situated factual content can be considered a useful tool to cope with these features.

Yet, as we reformulate Read's rejection of material consequence in informational terms, we do not directly object to the former consideration, as well as to its compliance with the DTL-intuition. Extending our language with the lattice-theoretical constant t, this concern can be given a more precise formulation. Defining the conditional content using the lattice conjunction, and the constant t as $s \Vdash t$ for all $s \in S$, we can exploit the material equivalence of A and $t \rightsquigarrow A$ (where $\rightsquigarrow$ is a lattice implication) as follows.

$$\begin{aligned} CONT_s(A \mid t) &= CONT_s(A \sqcap t) \setminus CONT_s(t) \\ &= (S \setminus \{s : s \Vdash A \sqcap t\}) \setminus (S \setminus \{s : s \Vdash t\}) \\ &= S \setminus \{s : s \Vdash A\} \\ &= CONT_s(A) \end{aligned}$$

This, in turn, contributes to a suitable perspective that gives situated factual content an interpretation in terms of material consequence: it connects situated factual content to derivability in an arbitrary situation. Unfortunately, derivability at an arbitrary situation is hardly better than truth. It does not give us a better grip on the logicality of $CONT_s$. As impossible situations cannot be excluded, nothing is true in all situations; derivability fails, exactly like truth does, to behave in a logically constrained way.

In the end, we do not quite see how this conclusion can be avoided. Even more, by separating the factual from the constraining content of a message we may have broken the previously existing connection between content-containment and content simpliciter. It is hard to see how we can at the same time draw a line between facts and constraints, and provide more than a fairly thin case for the logicality of the former. Such a thin case can take two forms. The first one, is to accept $CONT_s(A \mid B)$ as a degenerate, but still logical account of content-containment. This line is, given its dependence on the lattice-constant

t for connecting content and content-containment, not our preferred option (see also Paoli, 2006+).

The second case makes an appeal to the behaviour of $CONT_s$ with respect to, on the one hand, possible situations, and, on the other hand, logical situations. As mentioned, a situation is impossible just when it supports a contradiction. Conversely, a situation s is possible if it fails to support any contradiction; i.e. when it holds that $s \sqsubseteq s^*$. This, one could suggest, makes possible situations apt for logical behaviour: they never explicitly deny a classical theorem. However, as hinted upon in section 2, the set of possible situations is not upwardly closed under $\sqsubseteq$, and the property of not denying a classical theorem can therefore not be expressed as a proposition. This is exactly where Mortensen (1986) fails in restoring the material disjunctive syllogism using consistency as a premise (Mares, 2004: 183–4). In that respect, logical situations are more robust: the set *Log* is upwardly closed, and hence 'being logical' expresses a proposition. Thus, if 1 is the group-theoretical constant satisfied by all $s \in Log$, we know that if $s \Vdash 1$, $s' \Vdash 1$ for all $s \sqsubseteq s'$, and, as a consequence, we also know that if $s \in Log$, then $s = s^*$; persistently failing to deny a classical theorem just means asserting that theorem.[9]

On how this affects the notion of factual content, we can make the following comments. If s is excluded by A ($s \in CONT_s(A)$), and A is a classical theorem, then s is only persistently excluded if s is impossible. Generally, the possible situations individuated by $CONT_s(A)$ are just those situations that have not yet decided whether A, and might thus be extended either way: they only *weakly exclude* A. Put differently, $CONT_s(A)$ individuates those situations wherein one cannot soundly infer A on the basis of explicitly availably information only. For an arbitrary situation, it means that nothing at all can thus be inferred.

Alternatively intersecting $CONT_s(A)$ with the set of situations satisfying 1 (i.e. the logical situations), we can exploit the weak equivalence of A and $1 \rightsquigarrow A$ and show the duality of situated factual content and enthymematic derivability by assuming 1 to be a suppressed but obviously true antecedent (Anderson, Belnap & Dunn, 1992: 35; Paoli, 2002: 78–9).

$$\begin{aligned} CONT_s(A \mid 1) &= CONT_s(A \sqcap 1) \setminus CONT_s(1) \\ &= (S \setminus \{s : s \Vdash A \sqcap 1\}) \setminus (S \setminus \{s : s \Vdash 1\}) \\ &= \{s : s \Vdash 1\} \setminus \{s : s \Vdash A \sqcap 1\} \\ &= \{s : s = s^*\} \setminus \{s : s = s^* \ \&\ s \Vdash A\} \\ &= CONT_w(A) \end{aligned}$$

Since given the explicit assumption that 1, the situated factual content of A is just the worldly content of A, the robust logicality of $CONT_s$

cannot be regarded independently from $CONT_w$. At first sight, this is just additional evidence for Read's point of view: the classical account is predominant, and situated factual content is just a crippled classical account. By providing an independent reason of accepting the latter, the slide to worldly content is shown to be harmless for the independence of the situated account.

To motivate the respectability of a notion of worldly content, we need to sketch a situation where it provides the most accurate account of informational content. To do so, we rely on a strategy outlined in Allo (submitted) and provide a plausible interpretation of a worldly perspective. Let us go back to our initial example, and assume that right after the second move in the game was made (see figure 2), the sender wrote the message "P_3a_7 `or` $\sim P_3a_7$" on a slip of paper. Imagine that this message remained obscured to an hypothetical receiver until right after the game was duly completed. If so, one cannot but conclude that (no matter how the game was completed) the message is devoid of any content. However, since we could have come to the same conclusion using $CONT_c$, this is hardly sufficient to settle the correctness of $CONT_w$. The dispensability of $CONT_w$ is exactly what we could have expected on the basis of what many proponents of relevant logic have claimed: relevant logic is self-sufficient, it validates all the classical inferences while avoiding its confusions. What we do miss in using $CONT_c$ as an all-purpose measure, is the fact that from a worldly perspective the content conveyed by this message is as factual as it can be. More generally, in our use of $CONT_w$ we stress that, irrespectively of whether from a situated perspective a message conveys factual or constraining content, from a worldly perspective it unavoidably conveys both. What our notion of worldly content accounts for, is the additional consideration that, from a worldly perspective only, factual and constraining content actually do coincide. Specifically, their collapse is not a matter of equivocation, but of a lesser—yet perfectly adequate—discriminatory power: it does not rest on a confusion of factual and constraining content, but on their actual inseparability.

Here too, one can object that this collapse rests upon weak equivalences of lattice and group-disambiguations only. For both kinds of content to be equivalent, an additional truth must be assumed. But then, if the functioning of worldly content—including the collapse of facts and constraints—can be simulated by means of an additional premise, why include an independent account of worldly content? This question is answered in the next section as a reply to a monist objection due to Priest.

Priest's Objection

As anyone supporting the correctness of a sub-classical logic, both monists and pluralists face the problem of explaining the apparent validity of inferences their prefered logic deems invalid. Obviously, the monist cannot appeal to stronger but equally good logics to cope with this issue, and must therefore look for other solutions. It is very likely that, as Read shows, an appeal to unambiguous connectives can do most, if not all the work a monist requires of it. Yet, if our argument in favour of a less-discriminating (worldly) perspective has any ground, the monist must come up with additional methods to accommodate this last phenomenon. As suggested above, a standard monist strategy can consist in adding new, domain-specific premises.

> Now, just as an intuitionist may use what amounts to classical logic when reasoning about finite situations, so a paraconsistent logician may use what amounts to classical logic given appropriate information about the domain. For example, sufficient information is that for all α, $(\alpha \wedge \neg\alpha) \rightarrow \bot$, $(\ldots)$. (Priest, 2001b: 28)

Using a system weaker than Priest's preferred logic, the additional premise we must appeal to is the group-theoretical constant 1. Despite the apparent effectiveness of this approach, we do not think it can ultimately replace an appeal to worldly content. To that effect, we advance two arguments, a proof-theoretical and a semantic one.

The proof-theoretical argument rests upon the fact that to prove the equivalence of lattice and group-theoretical connectives, the inclusion of 1 does not help. Adding it as a premise does warrant that *weak derivability* is just fine, but what it cannot do, is make the structural rules required for a weak derivability relation obsolete.

Semantically, however, we have already seen that adding 1 as an antecedent suffices to get $CONT_s(A \mid 1) = CONT_w(A)$. Bridging the gap between constraining and worldly content, a restriction to logical constraints as well as the assumption that A does not contain any group-theoretical connectives is additionally required.

$$\begin{aligned} CONT_c(A \mid 1) &= 1_c^{\sqsubseteq} \setminus \{s : s \Vdash 1 \otimes A\} \\ &= \{s_i : \exists s(s \Vdash 1 \;\&\; \forall A(c \Vdash A \Rightarrow s_i \Vdash 1 \otimes A))\} \setminus \{s : s \Vdash 1 \otimes A\} \\ (\text{if } c \in Log) &= \{s : s = s^*\} \setminus \{s : s = s^* \;\&\; s \Vdash A\} \\ &= CONT_w(A) \end{aligned}$$

Still, the specific semantic argument we want to present, does not directly depend on these formal considerations, but remains closer to the interpretation we gave in terms of communicating agents. It crucially hangs on the specific way we flesh out the distinction between situated and worldly content.

For 1 to be considered an additional premise within a context of communication, it can only be introduced as an explicitly sent message. This, however, requires the sender to have sufficient information at its disposal to assert 1, and the receiver to have the ability to recognise it as such. As should be clear by now, neither of these conditions can actually be fulfilled. All the sender and the receiver can be said to hold, is the weaker *non-persistent* consistency-premise. Recasting Gillies' strategy of dealing with epistemic modals (2004; 2006), consistency is better considered the result of a successful test upon (or a global property of) one's actual state of information rather than a genuine piece of information itself. Quite like Mortensen's consistency premise, the result of such a test cannot be preserved in one's later states of information. As these specific limitations show that there is no reliable test for the satisfaction of 1, no message can reliably convey the information required to settle its truth either.

So far, this only establishes that for a situated agent no message can persistently convey the information that there exists a worldly perspective upon the context of communication it is part of. It nevertheless remains possible that if a worldly perspective exists, the correct way of assessing content and content-containment from that perspective crucially depends upon the availability of an additional premise. Upon closer inspection, we think this option should equally be dismissed. Surely, adding a premise apparently gets you right there, but it also fails to acknowledge an important difference between the situated and the worldly perspective. What we specifically need to point at, is the specific role played by the additional premise. That is, it explicitly states the information a situated agent requires to assess the content of a message *as if* it were evaluated from a worldly perspective. This strategy enables a situated receiver to simulate the worldly perspective by assuming 1 to be true, not by knowing it to be true. Put differently, there is a gap between correctly assuming that all the messages one receives originate from a single agent, and the fact that there is only one agent who sends messages.

The two main properties of this monist strategy, are now clear: the inclusion of the new premise proceeds explicitly, and by assumption only. But if this is all the monist can advance, the following dilemma is hardly avoidable. Either the monist cannot realy account for the worldly perspective, or otherwise informational pluralism cannot be avoided by the logical monist. The first half is fairly trivial: that there is a worldly perspective just means that 1 is true, not that it is merely assumed to be true. But then, if 1 can be true while none can explicitly be informed of its truth, it can only be concluded that $CONT_w$ correctly assesses the worldly content of a message because it implicitly

incorporates the truth of 1. Yet, this is exactly what the pluralist with respect to informational content claims.

Conclusion: A Realist's Pluralism

To conclude, we still have to explain what the *realism* in the commitment to pluralism stands for. Basically, if logic deals with inferences based on explicitly available information only, a monist can show that he accommodates for all extra-logical reasoning (most likely classical inference-rules that go beyond what can be achieved through disambiguation only) by the explicit addition of supplementary premises. Yet, if we speak of content and content-containment along realist lines, we often have to make an appeal to a relation of content-containment or nesting that the receiver of a message does not know of. In such cases, the relevant work cannot any longer be done by explicitly adding premises; only a logical solution is acceptable.

Specifically, this shows that the usual monist strategy of pretending that one only needs to pay attention to the premises no longer works if our focus is content, and content-containment. This insight, forms the basis of a realist commitment to logical pluralism.

Notes

[1] This accounts differs from Hanson's own, since he identifies the content of A with the consequence-set of A.

[2] Remark that the situated content of A ($\{s : s \nVdash A\}$) does not express a proposition, and hence is not a persistent kind of content. Namely, it does not hold that if $s \nVdash A$, then $s' \nVdash A$ for all $s \sqsubseteq s'$. Such issues regarding the non-persistence of properties should be kept in mind.

[3] A second, more complicated, aspect of the relevant recapture of *DS*, namely the admissibility of rule γ is left asside (Anderson & Belnap, 1975: 25).

[4] Remark in that light that rules (7) and (8) effectively turn some rules of the game into logical rules.

[5] As it is defined in the appendix, $c_1; c_2$ refers to the result of applying c_1 to c_2, that is $c_1 \sqsubseteq_{c_2} c_1; c_2$.

[6] Let $c = c_1; c_2$ where $c_1 \in \mathit{Log}$. Assume for reductio that $c \in \mathit{Log}$ too. By $c_1 \sqsubseteq_{c_2} c$, we have that $c_2 \sqsubseteq_{c_1} c$, and since $c_1 \in \mathit{Log}$ we also have that $c_2 \sqsubseteq c$. But then, given our assumption that $c \in \mathit{Log}$, it must at least hold that $c_2 \sqsubseteq c_2^*$. But since c_2 can be any element of S, the latter should not hold in general.

[7] Remember that our appeal to multiple agents is itself an artefact we use to account for the real phenomenon under consideration: the distributed nature of information (on that topic, see also Barwise & Seligman, 1997).

[8] As suggested by Greg Restall (pc), the model of section 4 might in a sense be closer to the approach in his "Modelling Truthmaking" (Restall, 2000).

[9] If $s \Vdash 1$ then s is consistent too, that is $s \sqsubseteq s^*$. Consequently, $s^* \Vdash 1$ holds too, and by the same token $s^* \sqsubseteq s^{**}$. Since $s = s^{**}$, $s^* \sqsubseteq s$, and hence $s = s^*$.

Appendix

Definition 1 (Routley-Meyer Frame) A Routley-Meyer frame is a structure $(S, Log, \sqsubseteq_-, *)$, where S is a set of situations, Log the subset of logical situations in S, $\sqsubseteq_-$ a ternary relation on S, and $*$ a unary relation on S. A partial order $\sqsubseteq$ on S is defined as $s_1 \sqsubseteq s_2 := \exists c \in Log \;\&\; s_1 \sqsubseteq_c s_2$, and can be understood as a refinement-relation or information-ordering. Its core property is the persistence it enforces:

$$\text{if } s_1 \sqsubseteq s_2 \;\text{ and }\; s_1 \Vdash p \text{ then } s_2 \Vdash p$$

Further properties of the ternary relation $\sqsubseteq_-$ are:

$$\begin{aligned} s_1 \sqsubseteq_c s_2 &\;\Rightarrow\; c \sqsubseteq_{s_1} s_2 \\ \exists s \in Log \;\&\; s_1 \sqsubseteq_{c_1} s \sqsubseteq_{c_2} s_2 &\;\Rightarrow\; \exists c \in Log \;\&\; s_1 \sqsubseteq_c s_2 \;\&\; c_1 \sqsubseteq_{c_2} c \\ s_1 \sqsubseteq_c s_2 \;\&\; c' \sqsubseteq c &\;\Rightarrow\; s_1 \sqsubseteq_{c'} s_2 \end{aligned}$$

While $*$ ought to comply with:

$$\begin{aligned} s_1 \sqsubseteq_c s_2 &\;\Rightarrow\; s_2^* \sqsubseteq_c s_1^* \\ s &\;=\; s^{**} \end{aligned}$$

$\sqsubseteq_-$ is asymmetric, transitive, but not reflexive (the reflexivity of $\sqsubseteq_c$ is equivalent to $c \sqsubseteq c^*$, which we do not have), $\sqsubseteq$ on the other hand, is reflexive too.
Furthermore, we introduce ; as a shorthand for channel-composition:

$$c = c_1; c_2 \quad \text{iff} \quad c_1 \sqsubseteq_{c_2} c$$

Definition 2 (Satisfaction) An evaluation $\Vdash$ for a standard language including both lattice and group-theoretical connectives is given by the following clauses:

$$\begin{aligned} s \Vdash A \sqcap B &\;\text{ iff }\; s \Vdash A \text{ and } s \Vdash B \\ s \Vdash A \sqcup B &\;\text{ iff }\; s \Vdash A \text{ or } s \Vdash B \\ s \Vdash A \rightsquigarrow B &\;\text{ iff }\; s^* \nVdash a \text{ or } s \Vdash B \end{aligned}$$

$$s \Vdash\!\sim A \;\text{ iff }\; s^* \nVdash A$$

$$\begin{aligned} s \Vdash A \to B &\;\text{ iff } \forall s_1, s_2 \in S \text{ where } s_1 \sqsubseteq_s s_2,\; s_1 \Vdash A \Rightarrow s_2 \Vdash B \\ s \Vdash A \oplus B &\;\text{ iff } \forall s_1, s_2 \in S \text{ where } s_1 \sqsubseteq_s s_2,\; s_1^* \nVdash A \Rightarrow s_2 \Vdash B \\ s \Vdash A \otimes B &\;\text{ iff } \exists s_1, s_2 \in S \text{ where } s_1 \sqsubseteq_{s_2} s,\; s_1 \Vdash A \;\&\; s_2 \Vdash B \end{aligned}$$

Proposition 2.1 (Compatibility and Consistency) The existence of inconsistent or impossible situations is warranted by the failure of $s \sqsubseteq s^*$:

$$s \Vdash p \sqcap {\sim}p \;\Rightarrow s \Vdash p \;\&\; s^* \nVdash p \;\text{ hence } s \not\sqsubseteq s^*$$

More generally, $\sqsubseteq$ and $*$ encode what it means for two situations s_1 and s_2 to be compatible as $s_1 \sqsubseteq s_2^*$, which is a symmetric relation.

$$\begin{aligned} \text{If } s_1 \sqsubseteq s_2^* \quad &\text{then} \quad s_2^{**} \sqsubseteq s_1^* \\ &\text{hence} \quad s_2 \sqsubseteq s_1^* \end{aligned}$$

This should not mean that two compatible states jointly obtain at a non-empty set of possible worlds. However, if both self-compatible, there is such a set of possible worlds.

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