Compressed Sensing mm-Wave SAR for Non-Destructive Testing Applications using Side Information

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Abstract—This paper evaluates the applicability of an innovative strategy for applying Compressed Sensing (CS) on Synthetic Aperture Radar (SAR) imaging, in the mm-wave range, using prior or structural side information. The studied technique adds the side information to the conventional CS minimization problem using an $l_1$-minimization approach, allowing for lower sub-Nyquist sampling than standard CS predicts. The applicability of this strategy on ultra-wideband SAR measurements is tested through simulations and real Non-Destructive Testing (NDT) experiments on a 3D-printed polymer object.

Index Terms—Compressed sensing (CS), Synthetic Aperture Radar (SAR), Non-Destructive Testing (NDT)

I. INTRODUCTION

Three-dimensional printing has become an extremely popular technology in many domains. This technology is widely used for prototyping, copying and producing complex objects [1]. Non-Destructive Testing (NDT) techniques are needed to image the inner structure of these objects in order to identify defects due to, for example, high temperature differences, calibration problems or unadapted printing speeds [2]. In this paper a SAR sensor is presented for imaging the interior of 3D-printed polymer objects.

Compressed Sensing (CS) was initially introduced to Synthetic Aperture Radar (SAR) applications for reducing the hardware complexity and storage needs [3], [4]. One of the requirements of compressed sensing is that the reconstructed image must be sparse in a given domain. In many SAR applications, this requirement is not met [5], thereby limiting the decrease of the undersampling bound. For 3D-printed objects additional side information can be added in the CS minimization equation, in order to overcome this problem: the exact physical dimensions and material properties of the designed object are known. The printed object can thus be compared with a similar printed object or with a simulated reference object. In this paper it is shown that adding the minimization of the difference between reference and real objects can severely decrease the number of needed samples.

The rest of the paper is organized as follows: Section 2 introduces briefly the theory of CS and the SAR sensor used in this work. In Section 3 a simulated sensor is presented and the impact of adding side information into the CS minimization equation is studied through simulations. Those results are then validated by means of real measurements on a 3D-printed object. Finally, conclusions are drawn and possible approaches for future work are presented.

II. BACKGROUND

A. Compressed Sensing

Suppose an unknown sparse signal $x \in \mathbb{C}^N$ with $k \ll N$ non-zero elements is sensed by applying $n$ linear measurements: $y = Ax$, where $A$ is a sensing matrix of size $n \times N$, satisfying the Restricted Isometry Property (RIP) [6]. Conventional compressed sensing [7] theory ensures an exact reconstruction of the signal $x$ in the case of severe undersampling ($n \ll N$) with high probability, by solving the following basis pursuit problem:

$$\min \|x\|_1 \quad \text{s.t.} \quad \|Ax - y\|_2 \leq \epsilon,$$  \hspace{1cm} (1)

where $\|\cdot\|_1, \|\cdot\|_2$ denote the $\ell_1$-norm and $\ell_2$-norm, respectively, and $\epsilon$ is an accuracy threshold.

The CS sub-sampling bound can be decreased even further if side information is available. Suppose $x^* \in \mathbb{C}^N$ to be a good estimate of $x$. The similarity between the side information and the unknown signal can be added to the minimization problem and equation (1) becomes:

$$\min \|x\|_1 + \beta \|g(x - x^*)\|_2 \quad \text{s.t.} \quad \|Ax - y\|_2 \leq \epsilon,$$  \hspace{1cm} (2)

with $\beta > 0$ and with $g$ a function expressing the similarity between $x$ and $x^*$. Popular choices for $g$ in literature are the $l_1$- and the $l_2$-norm, known as $l_1$-$l_1$ minimization and $l_1$-$l_2$ minimization respectively. In [8] and [9] both approaches are compared and the performance bounds are determined.
for Gaussian random sensing matrices. The authors conclude that, in general, as long as the similarity is significant, 1-1 minimization performs better than classic CS or the 1-2 minimization. The authors in [8] and [9] also prove, both theoretically and experimentally, that the sharpest bound is obtained for \( \beta = 1 \). Taking those results into account equation (2) becomes:

\[
\min \| x \|_1 + \| x - x^* \|_1 \quad \text{s.t.} \quad \| Ax - y \|_2 \leq \epsilon. \tag{3}
\]

In literature, 1-1 minimization has been applied in medical imaging: X-ray Computed Tomography (CT) [10] and Magnetic Resonance Imaging (MRI) [11], and for compressive video background subtraction [12] [13].

B. Stepped FMCW SAR

An ultra-wideband stepped Frequency-Modulated-Continuous Wave (FMCW) SAR was used for performing measurements. The parameters of this sensor were used to process both experimental and simulated data. The signals emitted by this type of sensor are expressed by:

\[
s(t) = \sum_{l=1}^{L} c \exp(j2\pi f_l t), \tag{4}
\]

with \( f_l = f_0 + (l - 1)\Delta f \) being the discrete frequencies, \( \Delta f \) being equal to \( B/L \) and \( f_0 \) equal to the starting frequency. \( B \) stands for the total emitted bandwidth and \( L \) equals the number of discrete frequencies. At reception, the echoes from \( K \) scatterers, with reflection coefficients \( a_{k,t} \) and at distances equal to \( r_k \) from the sensor, are demodulated, using a homodyne demodulation scheme and resulting in a beat signal:

\[
s_b(t) = \sum_{k=1}^{K} \sum_{l=1}^{L} a_{k,t} \exp \left( -j2\pi f_l \frac{2r_k}{c} \right). \tag{5}
\]

where \( c \) is the propagation speed of the electromagnetic waves through the medium (\( c = \frac{\lambda}{\sqrt{\mu \epsilon}} \)). A satisfying resolution in range (cross-track) and in azimuth (along-track) can be obtained, after applying a SAR compression algorithm (e.g.: [14] [15]) on this beat signal, due to the ultra-wide bandwidth and the synthetic aperture created by the relative motion between sensor and object. The range and azimuth resolutions can be estimated by equations (6) and (7), respectively:

\[
\Delta r_\text{ra} = \frac{c}{2B}, \tag{6}
\]

\[
\Delta r_\text{az} = \frac{c}{2f_0 \sin(\theta)}, \tag{7}
\]

where \( \theta \) is the opening angle of the antenna. Examples of the use of this type of sensor for NDT applications can be found in [16].

<table>
<thead>
<tr>
<th>Table 1: Specifications of the Sensor Used for the Simulated and Real Experiments</th>
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<tbody>
<tr>
<td><strong>NDT Sensor parameters</strong></td>
</tr>
<tr>
<td>Starting frequency ( f_0 )</td>
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<tr>
<td>Bandwidth</td>
</tr>
<tr>
<td>Frequency step ( \Delta f )</td>
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<tr>
<td>Scanning distance</td>
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<tr>
<td>Number of azimuth measurements</td>
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<tr>
<td>Number of frequencies</td>
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<tr>
<td>Aperture angle (-3 dB)</td>
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</tbody>
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Fig. 1. Experimental results obtained through simulation. 64 random reference scenes with a sparsity equal to 0.05 (top figure) and equal to 0.20 (second figure) were built. Each of them was changed with an increasing change rate going from 0.04 to 0.25. For each change rate the mean minimum number of samples (undersampling rate \( \delta \)) was determined for obtaining a correct reconstruction. This was done for the three different reconstruction approaches: (1) classic CS (in blue); (2) 1-1 minimization (in red) and (3) 1-2 minimization (in green).

III. Simulations

The parameters of the sensor used for the results obtained through simulations are identical to the real sensor and are listed in Table 1. These parameters are used to build the sensing matrix \( A \): each of the \( N \) columns of \( A \) corresponds to the simulated responses, and thus the beat signals, of a scene with a single point scatterer. These vectorized responses of length \( N \) are randomly sampled by selecting \( n \) rows of the sensing matrix. This corresponds to a random sampling over all the rows and columns of the raw data.

In order to experimentally evaluate the applicability of the 1-1 minimization approach on SAR data, random scenes of 1230 pixels are simulated. The reflection coefficients of those scenes are the side information \( x^s \), used for the reconstruction. Two sparsity rates are considered, namely, \( k/N = 0.05 \) and \( k/N = 0.20 \). The reflection coefficients of \( p \) random pixels of these reference scenes are then substituted by different random reflection coefficients. The simulated received signals from the adapted scenes are randomly sub-sampled with a varying undersampling rate \( \delta_\text{s} = n/N \in (0, 1) \). Fig. 1 shows the mean minimum undersampling rate necessary for a good reconstruction using the conventional CS approach, the 1-2 and the 1-1 minimization approaches for a varying change rate (= \( p/N \)). A reconstruction is considered to be
successful if the Peak Signal-to-Noise Ratio (PSNR) between the reconstructed scene and the adapted original scene $x$ is less than an arbitrarily chosen value of 20 dB.

Three reconstruction strategies are compared:

1. Conventional Compressed Sensing:
   \[ \min \| x \|_1 \quad \text{s.t.} \quad \| Ax - y \|_2 \leq \epsilon \]  

2. $l1-l2$ minimization:
   \[ \min \| x \|_1 + \frac{1}{2} \| x - x^* \|_2^2 \quad \text{s.t.} \quad \| Ax - y \|_2 \leq \epsilon \]  

3. $l1-l1$ minimization:
   \[ \min \| x \|_1 + \| x - x^* \|_1 \quad \text{s.t.} \quad \| Ax - y \|_2 \leq \epsilon \]

The results presented in Fig. 1 show that $l1-l1$ minimization performs drastically better than the classic CS as long as the similarity between the reference scene and the reconstructed scene is significant. The $l1-l1$ minimization also performs slightly better than the $l1-l2$ minimization. As can be expected intuitively, the positive impact of the use of side information becomes more important when the sparsity of the scene decreases. If the scene is already sparse (top graph of Fig. 1), the impact of the side information decreases rapidly for increasing changing rates. For a lower sparsity rate of the reference image (bottom graph of Fig. 1), the impact stays more important for increasing changing rates.

Further simulations are performed in order to evaluate the robustness against noise of the side-information CS algorithms compared to the conventional CS strategy. 64 random reference scenes with a sparsity equal to 0.20 were built. Each of them was changed with a change rate equal to 0.20. Noise is added to the measurements of the adapted scenes to obtain a varying Signal-to-Noise Ratio (SNR) from 0 to 28 dB. Fig. 2 depicts the mean minimum sub-sampling rate for a successful reconstruction for increasing SNR. A positive impact on the robustness against noise can be seen: the side-information algorithms allow for sub-sampling at lower SNR and the noiseless mean undersampling rates (the horizontal lines) are also reached at lower SNR compared to the conventional CS.

### IV. EXPERIMENTAL RESULTS ON REAL DATA

Only the results obtained with the $l1-l1$ minimization algorithm will be compared to the conventional CS approach, given the better performance of the $l1-l1$ minimization, as discussed in [8] and verified with the simulations presented in this document. The object under test (Fig. 2) is a 3D-printed rectangular block (dimensions: 18 cm in length x 5 cm in depth x 10 cm in height) made out of ABS (Acrylonitrile-Butadiene-Styrene) polymer, with three cylindrical holes (radius: 5 mm) reaching from top to bottom. This object is compared to an undamaged massive reference object made out of the same material and possessing the same dimensions.

The block is placed on a platform fixed on a scanning stage at a standoff distance of 27 cm from the antenna. Between each range measurement, the block is moved 1 cm and one complete measurement is composed of 30 azimuth measurements. Fig. 3 shows the compressed images in range and azimuth obtained from a measurement sampled at the Nyquist rate of the reference block (on the left) and the block with three defects (on the right). Further reconstructions will be compared with these images and expressed in PSNR (dB) in order to evaluate the reconstruction performance.

Fig. 4 depicts a visual comparison between the classic CS reconstructed images and the $l1-l1$ reconstruction for four different undersampling rates (80%, 60%, 40% and 20%). The $l1-l1$ performs remarkably better than the conventional reconstruction: with only 40% of the samples, the defects can still be detected, whereas even the front of the block cannot be identified with the classic approach. These results are of course subjected to the stochastic behavior of the results, due to the random sampling. Fig. 5 gives a statistical overview of the PSNR for 32 random sampling executions for both approaches and for the same four undersampling rates. Once again, a remarkable gain in reconstruction performance can be identified for the $l1-l1$ reconstruction compared to the classic CS reconstruction. The results obtained with the conventional CS approach are noticeably more spread out around the mean value, resulting in larger boxplots and differences between the extrema (Fig. 5).
Fig. 4. Reconstructed images from the fully sampled data. The left image is the image obtained for the reference block. The two interfaces (the front of the block at 0.27 m and the back of the block at 0.33 m) are clearly visible. The right image is obtained for the block with defects. The three defects are clearly visible and highlighted in green.

![Fig. 4.](image1)

Fig. 5. Visual comparison of the reconstructed images obtained by solving the classic CS minimization (left column) and obtained with the l1-l1 minimization approach (right column), for different sub-sampling rates (from top to bottom: 80%, 60%, 40%, 20%).

![Fig. 5.](image2)

Fig. 6. Statistical result for the reconstruction from 32 different random sampling executions with a subsampling rate equal to 20%, 40%, 60% and 80% of the Nyquist sampling rate. The l1-l1 minimization (light blue) clearly outperforms the conventional CS (in dark blue) approach. The quality of the reconstruction is compared to the reconstruction using 100% of the samples and expressed in PSNR [dB].

![Fig. 6.](image3)

V. CONCLUSIONS AND FUTURE WORK

This paper evaluates the applicability of adding side information to the CS minimization equation using an l1-l1 minimization for SAR imaging. Simulations and tests on real data prove the benefit by decreasing the number of samples, needed to obtain an accurate reconstruction, for this approach compared to conventional CS as long as the similarity with the reference image is acceptable. Simulations show that the l1-l1 minimization performs better than the l1-l2 minimization. Further sub-sampling would be possible if the non-sparse data were expressed in a sparser domain (e.g. wavelets) and with the use of a block sparsity algorithm.

REFERENCES


