Comparing Classical and Relativistic Kinematics in First-Order Logic

UNILOG 2018 Logic & Physics Session

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A *translation* is a function between formulas of languages preserving the logical connectives, i.e. $Tr(\phi \wedge \psi) = Tr(\phi) \wedge Tr(\psi)$, etc.

An *interpretation* of theory $\mathbf{Th_1}$ in theory $\mathbf{Th_2}$ is a translation Tr which translates all tautologies and all axioms of $\mathbf{Th_1}$ into theorems of $\mathbf{Th_2}$.

A *definitional equivalence* exists between two theories if those theories can be interpreted in each other and if all formulas from both theories translated into the other theory and back are logical equivalent to the original formulas.

We use these concepts to show the *differences*

between theories which are *not* equivalent.

There are translations Tr, Tr_+, and Tr'_+ between the languages of Classical Kinematics and Special Relativity Theory such that:

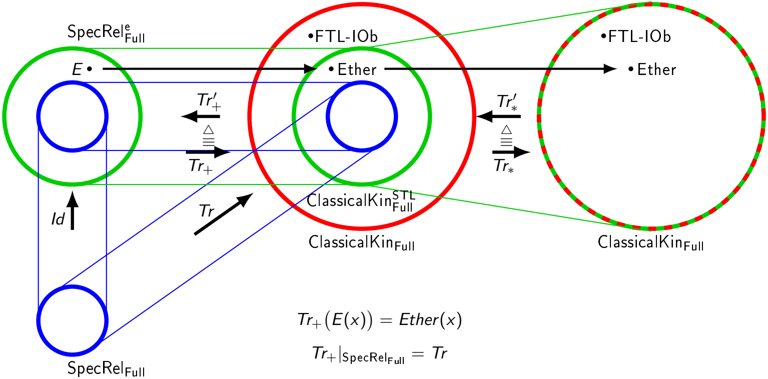
- $ClassicalKin_{Full} \vdash Tr(SpecRel_{Full})$
- $ClassicalKin^{STL}_{Full} \vdash Tr_+(SpecRel^e_{Full})$
- $SpecRel^e_{Full} \vdash Tr'_+(ClassicalKin^{STL}_{Full})$
- Definitional equivalence: $SpecRel^e_{Full} \stackrel{\triangle}{\equiv} ClassicalKin^{STL}_{Full}$, i.e., Tr_+ and Tr'_+ are inverses of each other up to logical equivalence in $SpecRel^e_{Full}$ and $ClassicalKin^{STL}_{Full}$.

There are translations Tr_*, and Tr'_* between the languages of Classical Kinematics and Classical Kinematics restricted to Slower-Than-Light oberservers such that:

- $ClassicalKin^{STL}_{Full} \vdash Tr_*(ClassicalKin_{Full})$
- $ClassicalKin_{Full} \vdash Tr'_*(ClassicalKin^{STL}_{Full})$
- Definitional equivalence: $ClassicalKin_{Full} \stackrel{\triangle}{\equiv} ClassicalKin^{STL}_{Full}$,

and hence by transitivity of definitional equivalence:

- $SpecRel^{e}_{Full} \stackrel{\triangle}{\equiv} ClassicalKin_{Full}$.



$$Tr_{+}(E(x)) = Ether(x)$$

$$Tr_{+}|_{\mathsf{SpecRel}_{\mathsf{Full}}} = Tr$$

$$\mathbf{Kin}{:=}\{\mathsf{AxEField}, \mathsf{AxEv}, \mathsf{AxSelf}, \mathsf{AxSymD}, \mathsf{AxLine}, \mathsf{AxTriv}, \mathsf{AxNoAcc}\}$$

$$\mathbf{ClassicalKin}_{\mathbf{\mathit{Full}}}{:=}\mathbf{Kin} \cup \{\mathsf{AxEther}, \mathsf{AbsTime}, \mathsf{AxThExp}_{+}\}$$

$$\mathbf{SpecRel}_{\mathbf{\mathit{Full}}}{:=}\ \mathbf{Kin} \cup \{\mathsf{AxPh}_{c}, \mathsf{AxThExp}\}$$

AxEField :

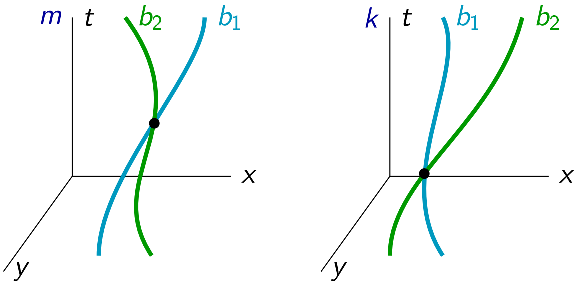
The **structure of quantities** $\langle Q, +, \cdot, \leq \rangle$ *is an Euclidean field,*

- Real numbers: $\mathbb{R}$,
- Real algebraic numbers: $\overline{\mathbb{Q}} \cap \mathbb{R}$,
- Hyperreal numbers: $\mathbb{R}^*$,
- Real constructable numbers,
- Etc...

AxEv :

Inertial observers coordinatize the same events (meetings of bodies).

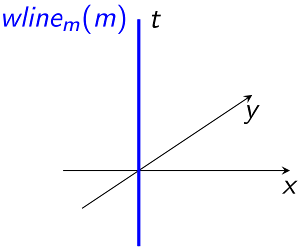
$$ev_m(\bar{x}) := \{b : W(m, b, \bar{x})\}$$



$$\forall m\, k\bar{x}\ \ [\mathsf{IOb}(m) \wedge \mathsf{IOb}(k) \rightarrow \exists \bar{y}\ \ ev_m(\bar{x}) = ev_k(\bar{y})].$$

AxSelf :

Every Inertial observer is stationary according to himself.

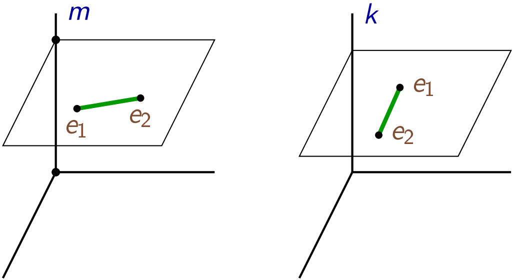


$$\forall mtxyz \ \Big(\mathsf{IOb}(m) \to \big[\mathsf{W}(m,m,t,x,y,z) \leftrightarrow x = y = z = 0\big]\Big).$$

AxSymD :

Inertial observers agree as to the *spatial distance* between two *events* if these two *events* are *simultaneous* for both of them.

$$\mathrm{space}(\bar{x},\bar{y}) := \sqrt{(x_2-y_2)^2+\ldots+(x_d-y_d)^2}$$



$$\forall mk\bar{x}\bar{y}\bar{x}'\bar{y}' \ \ [IOb(m) \land IOb(k) \land x_1 = y_1 \land x_1' = y_1' \land ev_m(\bar{x}) = ev_k(\bar{x}') \land ev_m(\bar{y}) = ev_k(\bar{y}') \to \mathrm{space}(\bar{x},\bar{y}) = \mathrm{space}(\bar{x}',\bar{y}')]$$

AxLine :

The worldlines of inertial observers are straight lines according to inertial observers.

$$\forall m k \bar{x}\bar{y}\bar{z}\ \big(IOb(m) \wedge IOb(k) \wedge W(m,k,\bar{x}) \wedge W(m,k,\bar{y}) \wedge W(m,k,\bar{z}) \\ \rightarrow \exists a\ [\bar{z}-\bar{x} = a(\bar{y}-\bar{x}) \vee \bar{y}-\bar{z} = a(\bar{z}-\bar{x})]\big).$$

(In ***SpecRel***, AxLine is a theorem, so including it as an axiom is redundant.)

AxTriv :

Any trivial transformation of an inertial observer is also an inertial observer.

$\forall T \in Triv[\forall m \exists k (\mathsf{w}_{mk} = T)]$, where Triv is the set of Trivial transformations, i.e. transformations that are isometries on space and translations on time.

$\forall T \in Triv$ may seem to be in second order logic. However, since a trivial transformation is nothing but an isometry on space (4×4 parameters) and a translation along the time axis (4 parameters), this is just an abbreviation for $\forall q_1, q_2, \ldots q_{20} \in \mathsf{Q}$ and together with $\mathsf{w}_{mk} = T$ a system of equations with 20 parameters in first order logic.

Let us define an *observer* as any body which can coordinatize other bodies:

$$Ob(k) \stackrel{def}{\Longleftrightarrow} \exists b \in \mathrm{B}, \exists \bar{x} \in Q^4 \big(\mathrm{W}(k, b, \bar{x})\big).$$

AxNoAcc :

All observers are inertial observers.

$$\forall k \in B[Ob(k) \rightarrow \mathrm{IOb}(k)].$$

AxAbsTime :

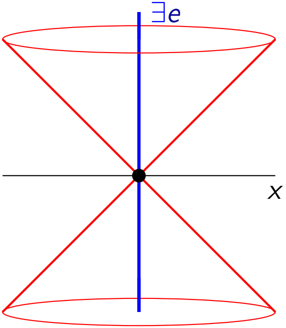
The time difference between two events is the same for all inertial observers.

$$\mathrm{time}(\bar{x},\bar{y}) := |x_1 - y_1|$$

$$\forall m k \bar{x} \bar{y} \bar{x}' \bar{y}' \; [IOb(m) \wedge IOb(k) \wedge ev_m(\bar{x}) = ev_k(\bar{x}') \wedge ev_m(\bar{y}) = ev_k(\bar{y}') \rightarrow \mathrm{time}(\bar{x},\bar{y}) = \mathrm{time}(\bar{x}',\bar{y}')].$$

AxEther(Einstein's AxLight) :

There exists an inertial observer in which the light cones are right.

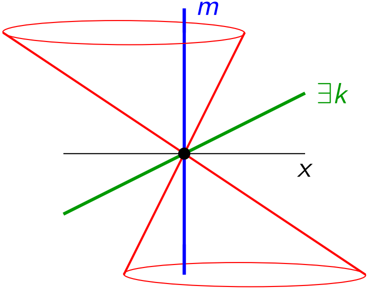


$$\exists ec\Big[\mathsf{IOb}(e) \wedge c > 0 \wedge \forall \bar{x}\bar{y}\Big(\exists p\big[\mathsf{Ph}(p) \wedge \mathsf{W}(e,p,\bar{x}) \wedge \mathsf{W}(e,p,\bar{y})\big] \leftrightarrow \mathsf{space}(\bar{x},\bar{y}) = c \cdot \mathsf{time}(\bar{x},\bar{y})\Big)\Big]$$

Note: in all figures, the speed of light $c = 1$.

AxThExp$_+$:

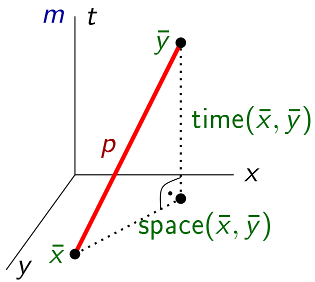
Inertial observers can move along any *non-horizontal straight line.*



$$\exists h \in B\big[IOb(h)\big] \land$$
$$\forall k \in IOb\,\forall \bar{x}, \bar{y} \in Q^4\ \big(x_0 \neq y_0 \rightarrow \exists k' \in IOb\big[\bar{x}, \bar{y} \in wl_k(k')\big]\big)\,.$$

AxPh_c :

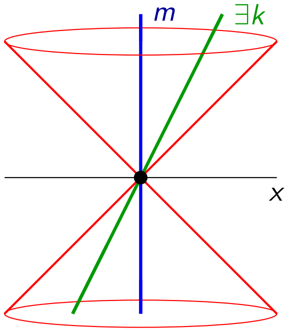
For any inertial observer, the speed of light is the same in every direction everywhere, and it is finite. Furthermore, it is possible to send out a light signal in any direction.



$$\exists c\Big[c > 0 \land \forall m\bar{x}\bar{y}\Big(\mathsf{IOb}(m) \to \exists p\big[\mathsf{Ph}(p) \land \mathsf{W}(m,p,\bar{x}) \land \mathsf{W}(m,p,\bar{y})\big] \leftrightarrow \mathsf{space}(\bar{x},\bar{y}) = c \cdot \mathsf{time}(\bar{x},\bar{y})\Big)\Big]$$

AxThExp :

Inertial observers can move with any speed slower than that of light.



$$\exists h\ \ [IOb(h)] \land \forall m\bar{x}\bar{y}\ \ \big(IOb(m) \land \mathrm{space}(\bar{x},\bar{y}) < c \cdot \mathrm{time}(\bar{x},\bar{y})$$
$$\rightarrow \exists k\ \ [IOb(k) \land W(m,k,\bar{x}) \land W(m,k,\bar{y}) \land m \uparrow k]\big).$$

$$\textbf{Kin}{:=}\{\mathsf{AxEField}, \mathsf{AxEv}, \mathsf{AxSelf}, \mathsf{AxSymD}, \mathsf{AxLine}, \mathsf{AxTriv}, \mathsf{AxNoAcc}\}$$

$$\textbf{ClassicalKin}_{\textit{Full}}{:=}\textbf{Kin} \cup \{\mathsf{AxEther}, \mathsf{AbsTime}, \mathsf{AxThExp}_{+}\}$$

$$\textbf{SpecRel}_{\textit{Full}}{:=}\ \textbf{Kin} \cup \{\mathsf{AxPh}_{c}, \mathsf{AxThExp}\}$$

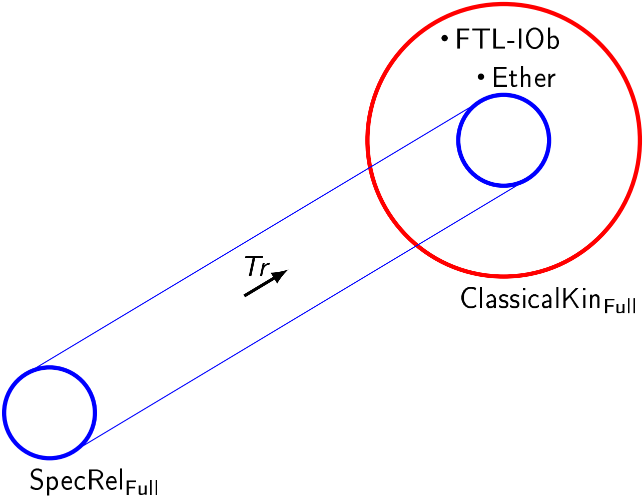
Justification Theorems:

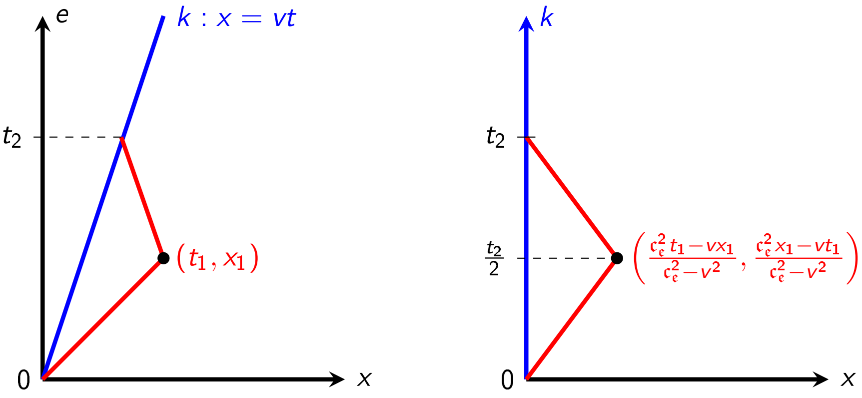
$\textbf{ClassicalKin}_{\textit{Full}} \vdash$ Worldview transformations are Galilean transformations.

$\textbf{SpecRel}_{\textit{Full}} \vdash$ Worldview transformations are Poincaré transformations
(see Theorem 2.1, p.639 in "A logic road from special relativity to general relativity" by H. Andréka, J. X. Madarász, I. Németi, and G. Székely, in *Synthese*, Vol. 186, Nr. 3, pages 633–649, 2012).

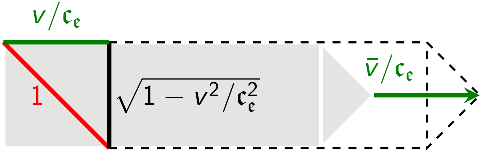
Theorem:

There is an interpretation Tr of **SpecRel_{Full}** *in* **$\text{ClassicalKin}_{Full}$**.

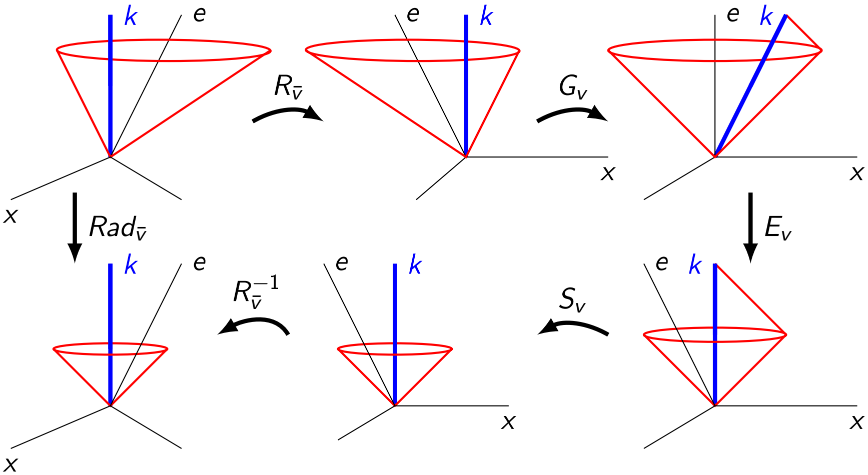




$$E_v := \begin{bmatrix} \frac{1}{1-v^2/\mathfrak{c}_e^2} & \frac{-v/\mathfrak{c}_e^2}{1-v^2/\mathfrak{c}_e^2} & 0 & 0 \\ \frac{-v}{1-v^2/\mathfrak{c}_e^2} & \frac{1}{1-v^2/\mathfrak{c}_e^2} & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{1-v^2/\mathfrak{c}_e^2}} & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{1-v^2/\mathfrak{c}_e^2}} \end{bmatrix}$$



$$S_v := \sqrt{1 - v^2/\mathfrak{c}_\mathfrak{e}^2}$$



$$Rad_{\bar v} = R_{\bar v}^{-1} \circ S_v \circ E_v \circ G_v \circ R_{\bar v}$$

$$Tr(a+b=c) \stackrel{\text{def}}{\equiv} a+b=c$$

$$Tr(a\cdot b=c) \stackrel{\text{def}}{\equiv} a\cdot b=c$$

$$Tr(a<b) \stackrel{\text{def}}{\equiv} a<b$$

$$Tr(\mathsf{W}^{SR}(k,b,t,x,y,z)) \stackrel{\text{def}}{\equiv}$$
$$\exists t'x'y'z'[\mathsf{W}^{CK}(k,b,t',x',y',z') \wedge Rad_k(t',x',y',z')=(t,x,y,z)]$$

$$Tr(\mathsf{IOb}^{SR}(k)) \stackrel{\text{def}}{\equiv} \mathsf{IOb}^{CK}(k) \wedge \forall e[Ether(e) \rightarrow speed_e^{CK}(k) < \mathfrak{c}_{\mathfrak{e}}]$$

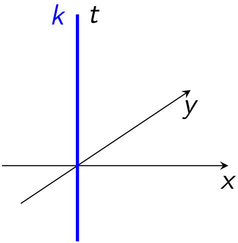
where

$$Ether(e) \stackrel{def}{\Longleftrightarrow} \mathsf{IOb}^{CK}(e) \wedge \forall p[Ph(p) \rightarrow speed_e^{CK}(p) = \mathfrak{c}_{\mathfrak{e}}]$$

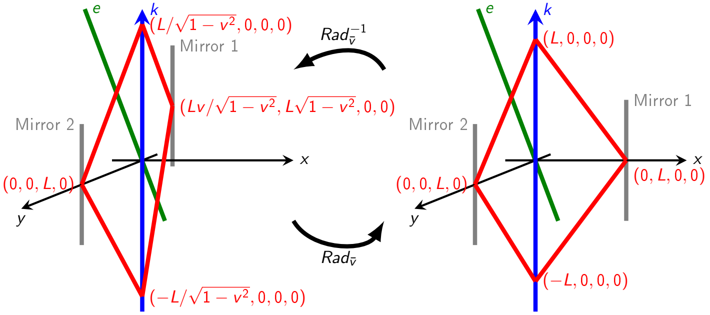
Example: Translation of AxSelf^{SR}

$$Tr[\forall k\ \ \mathsf{IOb}^{SR}(k) \rightarrow \forall \bar{x}\ \left(\mathsf{W}^{SR}(k,k,\bar{x}) \leftrightarrow x_2 = x_3 = x_4 = 0\right)]$$

$$\equiv \forall k\ \ \left[\mathsf{IOb}^{CK}(k) \wedge \forall e[Ether(e) \rightarrow speed_e^{CK}(k) < \mathfrak{c}_{\mathfrak{e}}]\right.$$
$$\left.\rightarrow \forall \bar{x}\ \left(\exists \bar{y}[\mathsf{W}^{CK}(k,k,\bar{y}) \wedge Rad_k(\bar{y}) = \bar{x}] \leftrightarrow x_2 = x_3 = x_4 = 0\right)\right]$$



Example: the Michelson–Morley Experiment



Note that while our translation function Tr translates axioms of special relativity theory into theorems of classical kinematics, models are transformed the other way round from classical mechanics to special relativity theory.

Theorem:

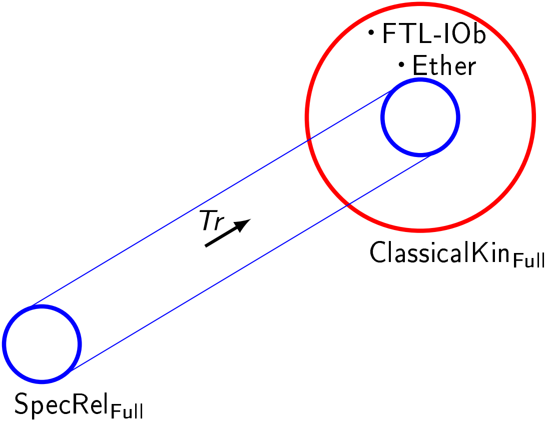
There is an interpretation Tr of **SpecRel**$_{Full}$ *in* **ClassicalKin**$_{Full}$.

$$\textbf{ClassicalKin}_{Full} \vdash Tr(\textbf{SpecRel}_{Full})$$

- **ClassicalKin**$_{Full}$ $\vdash Tr(\mathrm{AxEField}^{SR})$
- **ClassicalKin**$_{Full}$ $\vdash Tr(\mathrm{AxEv}^{SR})$
- **ClassicalKin**$_{Full}$ $\vdash Tr(\mathrm{AxSelf}^{SR})$
- **ClassicalKin**$_{Full}$ $\vdash Tr(\mathrm{AxSymD}^{SR})$
- **ClassicalKin**$_{Full}$ $\vdash Tr(\mathrm{AxLine}^{SR})$
- **ClassicalKin**$_{Full}$ $\vdash Tr(\mathrm{AxTriv}^{SR})$
- **ClassicalKin**$_{Full}$ $\vdash Tr(\mathrm{AxNoAcc}^{SR})$
- **ClassicalKin**$_{Full}$ $\vdash Tr(\mathrm{AxPh}_c)$
- **ClassicalKin**$_{Full}$ $\vdash Tr(\mathrm{AxThExp})$

Theorem:

There is no definitional equivalence between $\mathbf{SpecRel}_{Full}$ *and* $\mathbf{ClassicalKin}_{Full}$.



$Conceptual\_distance(\mathbf{SpecRel}_{Full}, \mathbf{ClassicalKin}_{Full}) > 0$

$\text{ClassicalKin}^{STL}_{Full} :=$ $\text{Kin} \cup \{\text{AxEther}, \text{AbsTime}, \text{AxThExp}^{\text{STL}}, \text{AxNoFTL}\}$

AxNoFTL :

All inertial observers move slower than light with respect to the ether frames.

$$\neg \exists m \big(IOb(m) \land \exists e [Ether(e) \land Speed^{CK}_{e}(m) \geq \mathfrak{c}_{\mathfrak{e}}] \big).$$

AxThExp$^{\text{STL}}$:

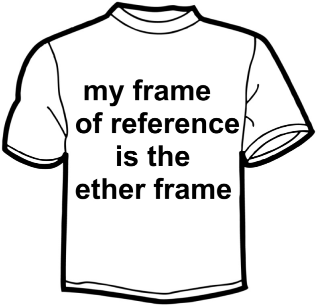
Inertial observers can move with any speed which is in the ether frame slower than that of light.

$$\exists h \ \big(IOb(h) \big) \land \forall e \bar{x} \bar{y} \big(Ether(e) \land \text{space}(\bar{x}, \bar{y}) < \mathfrak{c}_{\mathfrak{e}} \cdot \text{time}(\bar{x}, \bar{y}) \rightarrow \exists k \ \ IOb(k) \land W(e, k, \bar{x}) \land W(e, k, \bar{y}) \big).$$

$\mathbf{SpecRel}^{e}_{Full} := \mathbf{SpecRel}_{Full} \cup \{\text{AxPrimitiveEther}\}$

AxPrimitiveEther :

There is a non-empty class of ether observers, stationary with respect to each other, which is closed under trivial transformations.



$$\exists e\big(E(e) \land \forall k\big[[IOb(k) \land (\exists T \in Triv) w^{SR}_{ek} = T] \leftrightarrow E(k)\big]\big)$$

Theorem:

There is an interpretation Tr'_+ *of* $\mathsf{ClassicalKin}^{STL}_{Full}$ *in* $\mathsf{SpecRel}^{e}_{Full}$.

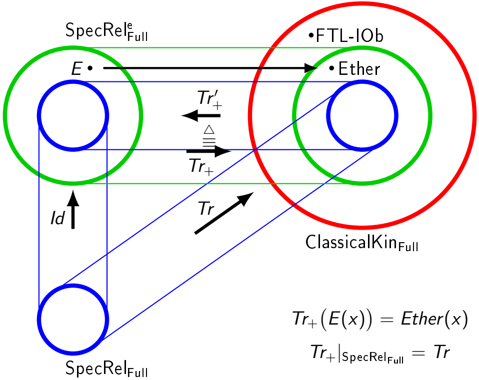
$$\mathsf{SpecRel}^{e}_{Full} \vdash Tr'_+(\mathsf{ClassicalKin}^{STL}_{Full})$$

- $\mathsf{SpecRel}^{e}_{Full} \vdash Tr'_+(\mathsf{AxEField}^{\mathsf{CK}})$
- $\mathsf{SpecRel}^{e}_{Full} \vdash Tr'_+(\mathsf{AxEv}^{\mathsf{CK}})$
- $\mathsf{SpecRel}^{e}_{Full} \vdash Tr'_+(\mathsf{AxSelf}^{\mathsf{CK}})$
- $\mathsf{SpecRel}^{e}_{Full} \vdash Tr'_+(\mathsf{AxSymD}^{\mathsf{CK}})$
- $\mathsf{SpecRel}^{e}_{Full} \vdash Tr'_+(\mathsf{AxLine}^{\mathsf{CK}})$
- $\mathsf{SpecRel}^{e}_{Full} \vdash Tr'_+(\mathsf{AxTriv}^{\mathsf{CK}})$
- $\mathsf{SpecRel}^{e}_{Full} \vdash Tr'_+(\mathsf{AxNoAcc}^{\mathsf{CK}})$
- $\mathsf{SpecRel}^{e}_{Full} \vdash Tr'_+(\mathsf{AxEther})$
- $\mathsf{SpecRel}^{e}_{Full} \vdash Tr'_+(\mathsf{AxAbsTime})$
- $\mathsf{SpecRel}^{e}_{Full} \vdash Tr'_+(\mathsf{AxThExp}^{\mathsf{STL}})$
- $\mathsf{SpecRel}^{e}_{Full} \vdash Tr'_+(\mathsf{AxNoFTL})$

Theorem:

Tr_+ and Tr'_+ are a definitional equivalence between $\mathbf{SpecRel}^{e}_{Full}$ *and* $\mathbf{ClassicalKin}^{STL}_{Full}$.

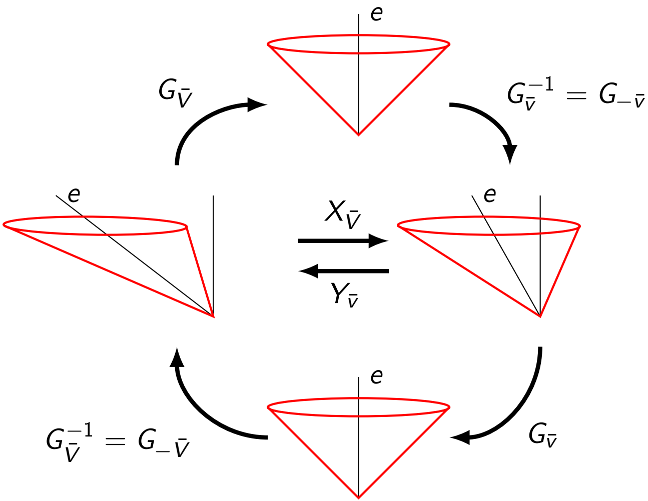
$Tr'_+ \circ Tr_+(\mathbf{SpecRel}^{e}_{Full}) \Leftrightarrow \mathbf{SpecRel}^{e}_{Full}$ and $Tr_+ \circ Tr'_+(\mathbf{ClassicalKin}^{STL}_{Full}) \Leftrightarrow \mathbf{ClassicalKin}^{STL}_{Full}$



$$Tr_+(E(x)) = Ether(x)$$

$$Tr_+|_{\mathrm{SpecRel_{Full}}} = Tr$$

$Conceptual\_distance(\mathbf{SpecRel}_{Full}, \mathbf{ClassicalKin}_{Full}) \leq 2$



$$\bar{v} = \frac{\mathfrak{c}_\mathfrak{e} \bar{V}}{1 + |\bar{V}|} \qquad \bar{V} = \frac{\bar{v}}{\mathfrak{c}_\mathfrak{e} - |\bar{v}|}$$

Theorem:

There is an interpretation Tr_* *of* $\mathsf{ClassicalKin}^{STL}_{Full}$ *in* $\mathsf{ClassicalKin}_{Full}$.

$$\mathsf{ClassicalKin}_{Full} \vdash Tr_*(\mathsf{ClassicalKin}^{STL}_{Full})$$

- $\mathsf{ClassicalKin}_{Full} \vdash Tr_*(\mathsf{AxEField}^{\mathsf{STL}})$
- $\mathsf{ClassicalKin}_{Full} \vdash Tr_*(\mathsf{AxEv}^{\mathsf{STL}})$
- $\mathsf{ClassicalKin}_{Full} \vdash Tr_*(\mathsf{AxSelf}^{\mathsf{STL}})$
- $\mathsf{ClassicalKin}_{Full} \vdash Tr_*(\mathsf{AxSymD}^{\mathsf{STL}})$
- $\mathsf{ClassicalKin}_{Full} \vdash Tr_*(\mathsf{AxLine}^{\mathsf{STL}})$
- $\mathsf{ClassicalKin}_{Full} \vdash Tr_*(\mathsf{AxTriv}^{\mathsf{STL}})$
- $\mathsf{ClassicalKin}_{Full} \vdash Tr_*(\mathsf{AxNoAcc}^{\mathsf{STL}})$
- $\mathsf{ClassicalKin}_{Full} \vdash Tr_*(\mathsf{AxEther}^{STL})$
- $\mathsf{ClassicalKin}_{Full} \vdash Tr_*(\mathsf{AxAbsTime}^{STL})$
- $\mathsf{ClassicalKin}_{Full} \vdash Tr_*(\mathsf{AxThExp}^{\mathsf{STL}})$
- $\mathsf{ClassicalKin}_{Full} \vdash Tr_*(\mathsf{AxNoFTL})$

Theorem:

There is an interpretation Tr'_* *of* ClassicalKin$_{Full}$ *in* ClassicalKin$^{STL}_{Full}$.

$$\textit{ClassicalKin}^{STL}_{Full} \vdash Tr'_*(\textit{ClassicalKin}_{Full})$$

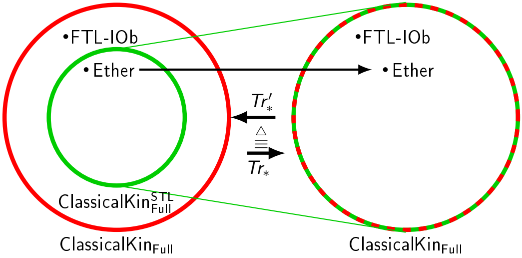
- ClassicalKin$^{STL}_{Full}$ $\vdash Tr'_*(\text{AxEField}^{CK})$
- ClassicalKin$^{STL}_{Full}$ $\vdash Tr'_*(\text{AxEv}^{CK})$
- ClassicalKin$^{STL}_{Full}$ $\vdash Tr'_*(\text{AxSelf}^{CK})$
- ClassicalKin$^{STL}_{Full}$ $\vdash Tr'_*(\text{AxSymD}^{CK})$
- ClassicalKin$^{STL}_{Full}$ $\vdash Tr'_*(\text{AxLine}^{CK})$
- ClassicalKin$^{STL}_{Full}$ $\vdash Tr'_*(\text{AxTriv}^{CK})$
- ClassicalKin$^{STL}_{Full}$ $\vdash Tr'_*(\text{AxNoAcc}^{CK})$
- ClassicalKin$^{STL}_{Full}$ $\vdash Tr'_*(\text{AxEther}^{CK})$
- ClassicalKin$^{STL}_{Full}$ $\vdash Tr'_*(\text{AxAbsTime}^{CK})$
- ClassicalKin$^{STL}_{Full}$ $\vdash Tr'_*(\text{AxThExp}_{+})$

Theorem:

Tr_* and Tr'_* are a definitional equivalence between **$\mathsf{ClassicalKin}^{STL}_{Full}$** *and* **$\mathsf{ClassicalKin}_{Full}$**.

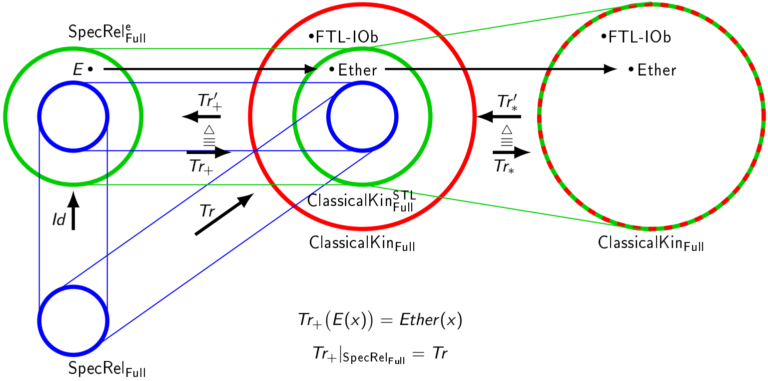
$Tr'_* \circ Tr_*(\mathsf{ClassicalKin}^{STL}_{Full}) \Leftrightarrow \mathsf{ClassicalKin}^{STL}_{Full}$ and

$Tr_* \circ Tr'_*(\mathsf{ClassicalKin}_{Full}) \Leftrightarrow \mathsf{ClassicalKin}_{Full}$



Theorem:

$Tr_* \circ Tr_+$ and $Tr'_+ \circ Tr'_*$ are a definitional equivalence between **$\mathsf{SpecRel}^e_{Full}$** *and* **$\mathsf{ClassicalKin}_{Full}$**.



$Conceptual\_distance(\mathbf{SpecRel}_{Full}, \mathbf{ClassicalKin}_{Full}) = 1$

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Koen Lefever and Gergely Székely
"Comparing Classical and Relativistic Kinematics in First-Order Logic"
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