

Nonparametric density and regression estimation has been the subject of intense investigation for many years and this has led to a large number of methods. One very well-known and commonly used class of estimators consists of the so-called *kernel-type estimators*, which are frequently used to estimate densities and regression functions. A typical kernel-type estimator based upon i.i.d. variables  $(X_1, Y_1), \dots, (X_n, Y_n)$  with values in  $\mathbb{R}^d \times \mathbb{R}^f$  is defined as  $\hat{\varphi}_{n,h}(t) = (nh^d)^{-1} \sum_{i=1}^n \varphi(Y_i)K((t - X_i)/h)$ , where  $K$  is a kernel function,  $0 < h < 1$  a bandwidth, and  $\varphi : \mathbb{R}^f \rightarrow \mathbb{R}$  is a suitable measurable function.

Although there are basically no restrictions on the choice of the kernel, the choice of the bandwidth is more problematic, as it is responsible for an important bias-variance trade-off of the resulting kernel-type estimator. One thus has to find an appropriate bandwidth that would lead to an estimator having a good balance between bias and variance. Typically, the bandwidth that is most appropriate will vary according to the situation and will depend on the available data. This means that one can no longer investigate the behavior of such "optimal" estimators based upon data-dependent bandwidth sequences via the classical results for estimators based upon deterministic bandwidth sequences.

The main purpose of this thesis is to prove "uniform in bandwidth" results for a wide variety of kernel-type estimators, meaning that a supremum over suitable ranges of bandwidths is added to the original asymptotic result. This extra supremum permits to handle kernel-type estimators based upon bandwidths that are functions of the data. Our methodology relies mainly on the theory of empirical processes, and the basic tools are appropriate exponential deviation inequalities and moment inequalities for empirical processes. Throughout the different chapters we will apply this method several times to establish the uniform in bandwidth consistency of specific classes of kernel-type estimators and conditional  $U$ -statistics.