Finite Geometry and Friends

Small weight code words

in the code of points and hyperplanes of PG(n, q)

Lins Denaux

Joint work with S. Adriaensen, L. Storme and Zs. Weiner

19th of June 2019

GHENT UNIVERSITY

1 Preliminaries

The code $C_{n-1}(n, q)$

Vector space over $\mathbb{F}_p$ spanned by the rows of the incidence matrix of hyperplanes and points in $\mathrm{PG}(n, q)$. Vectors = '**code words**'.

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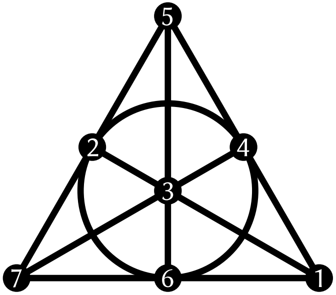
$$\text{hyperplanes}\left\{\overbrace{\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}}^{\text{points}}\right.$$



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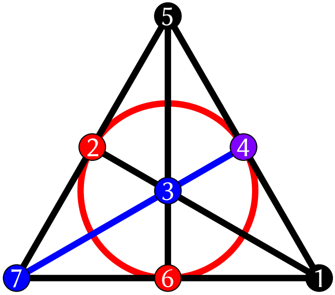
points

hyperplanes

1	1	1	0	0	0	0
1	0	0	1	1	0	0
1	0	0	0	0	1	1
0	1	0	1	0	1	0
0	1	0	0	1	0	1
0	0	1	1	0	0	1
0	0	1	0	1	1	0



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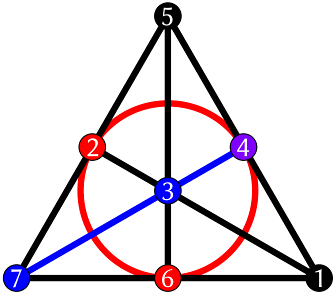
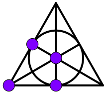
$$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ \color{red}0 & \color{red}1 & \color{red}0 & \color{red}1 & \color{red}0 & \color{red}1 & \color{red}0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ \color{blue}0 & \color{blue}0 & \color{blue}1 & \color{blue}1 & \color{blue}0 & \color{blue}0 & \color{blue}1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$



red + blue = (0 1 1 0 0 1 1) = 

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Vector space over $\mathbb{F}_p$ spanned by the hyperplanes as 0-1 incidence functions of the point set of PG(n,q). Functions = ‘**code words**’.

points

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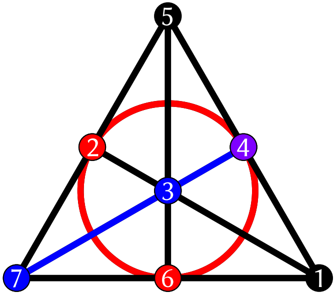
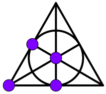
$$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$



red + blue = $(0\ 1\ 1\ 0\ 0\ 1\ 1)$ = 

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Small weight code words $\approx$ **few** hyperplanes (= lines)?

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Characterised up till $\mathrm{wt}(c) \leqslant 4q - 22$ (Szőnyi & Weiner):

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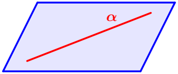

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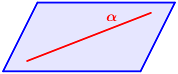

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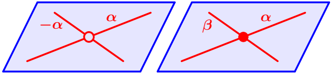

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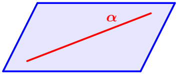

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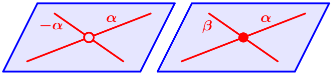

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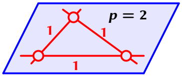

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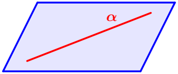

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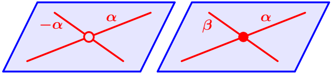

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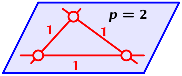

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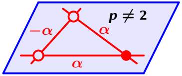

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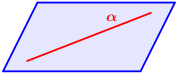

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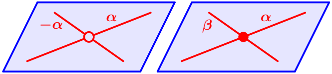

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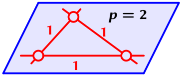

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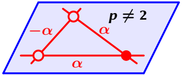
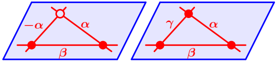


$\mathbf{wt(c) = 3q\,(\pm 1)}$



wt(c)= $3q - 1$ wt(c)= $3q$

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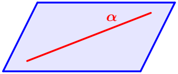

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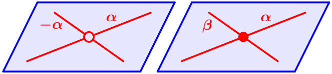

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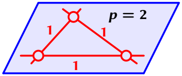

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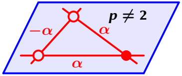

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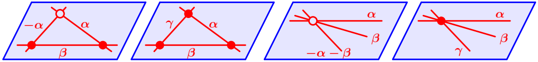

$\mathrm{wt}(c) = 3q - 1$ | $\mathrm{wt}(c) = 3q$ | $\mathrm{wt}(c) = 3q$ | $\mathrm{wt}(c) = 3q + 1$

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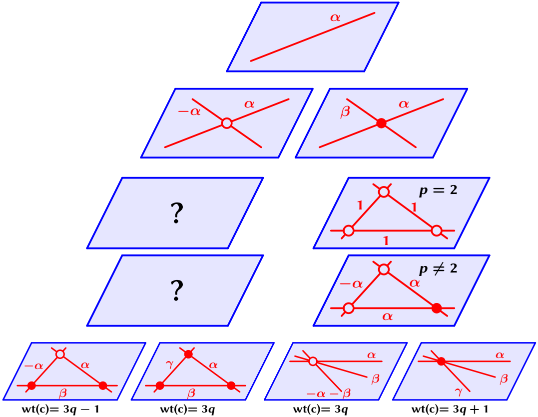

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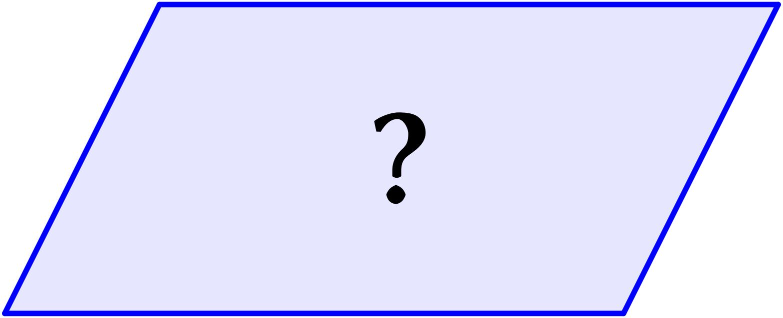
An ‘odd’ code word for $q = p$ (Bagchi; De Boeck & Vandendriessche):



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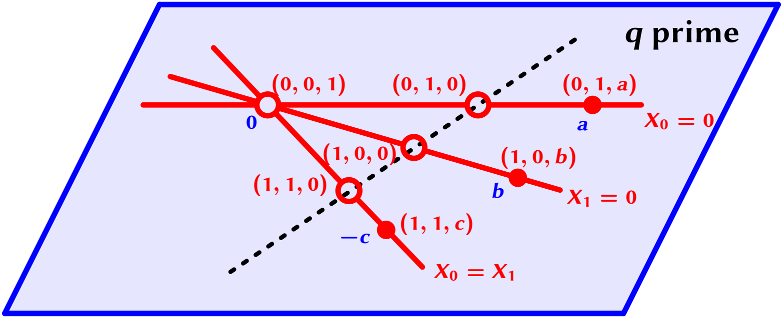

Proposition

- $\mathbf{wt}(\boldsymbol{c}) = \mathbf{3}\boldsymbol{q} - \mathbf{3}$, every $(2/3)$-secant $\rightarrow \alpha + \beta\,(+\gamma) = 0$.

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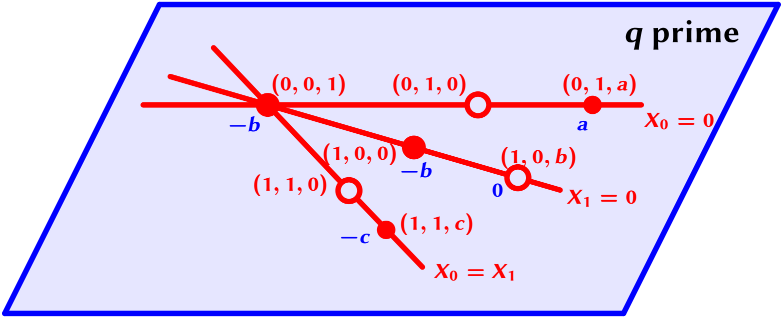

Proposition

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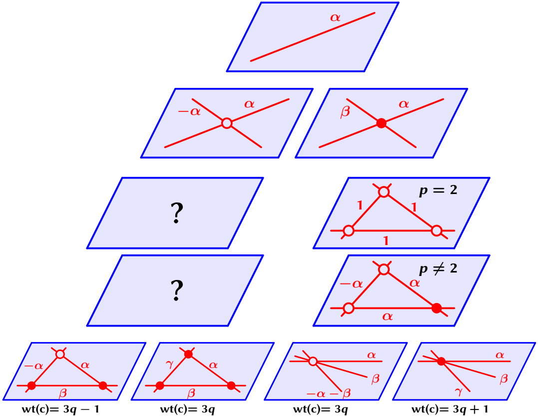

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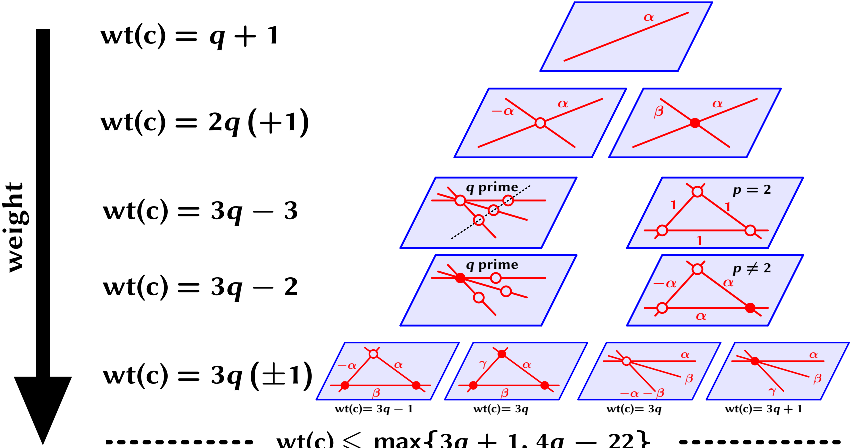

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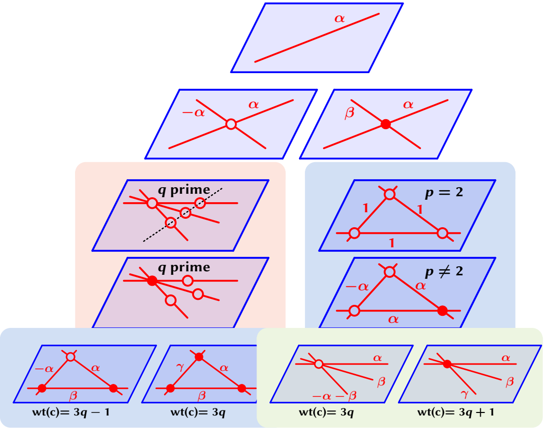

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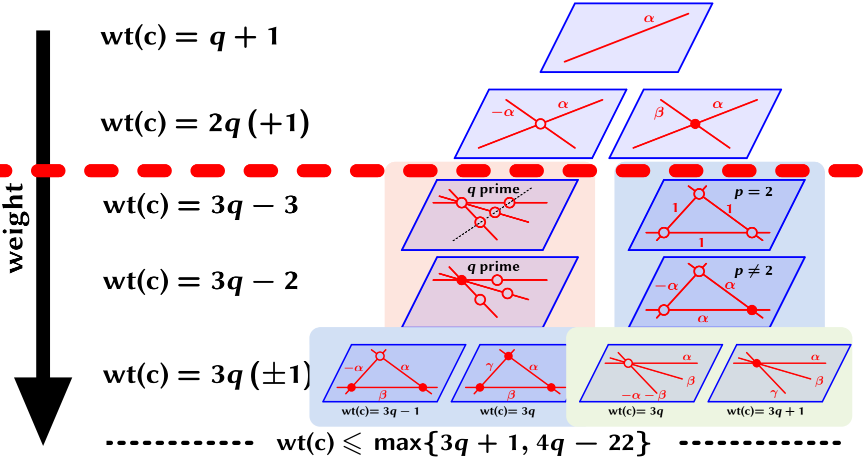

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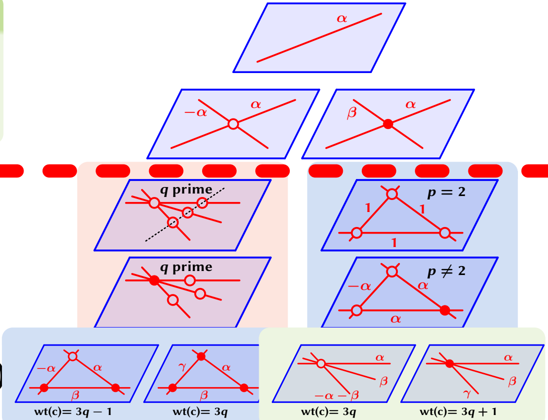

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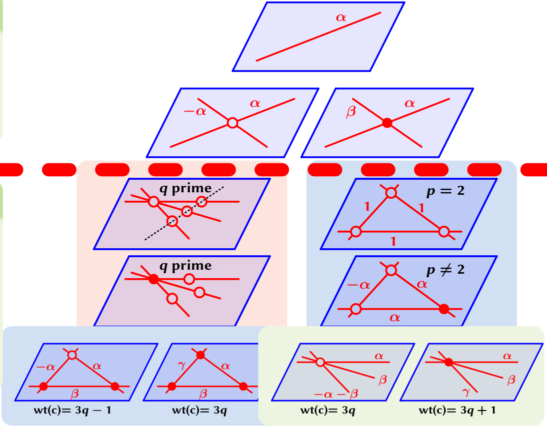
We'll focus on this bit



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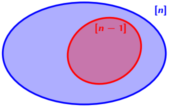
Smallest weight code words of $C_{n-1}(n, q)$: **generally known**.

$$\mathbf{wt(c)} = q^{n-1} + \cdots + q + 1$$



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Second smallest weight:
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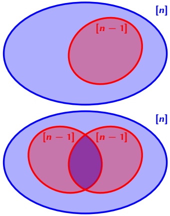

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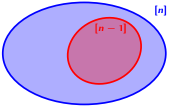

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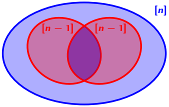
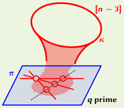


Our result: classification of next weights

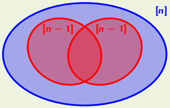
$$\mathbf{wt(c)} \lesssim 4q^{n-1} - \sqrt{8q} \cdot q^{n-2}$$



A quiet moment to think things through

First result: classification of the third smallest weight

$$\mathrm{wt}(c) = 2q^{n-1} + \cdots + q + 1$$



for all c with $2q^{n-1} < wt(c) \lesssim 3q^{n-1} - 6q^{n-2}$.

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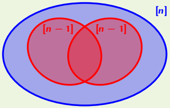

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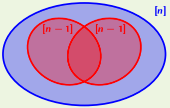

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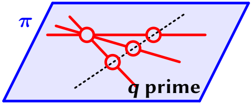

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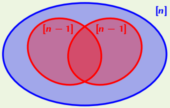

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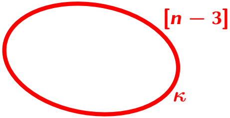

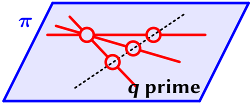

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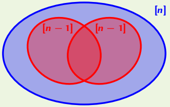

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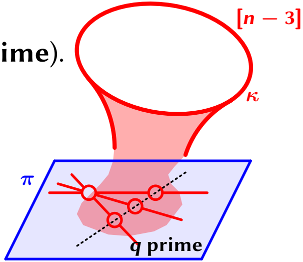

If $c = \sum_i \alpha_i l_i$, then $c' := \sum_i \alpha_i \langle l_i, \kappa \rangle$ is a linear combination of hyperplanes;
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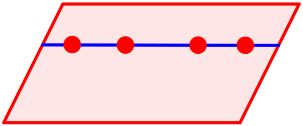

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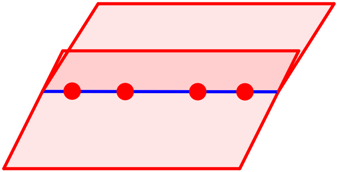

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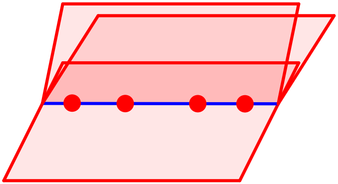

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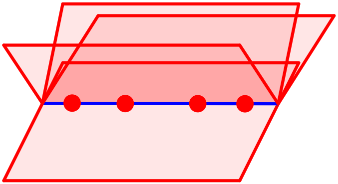

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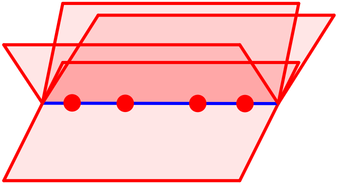

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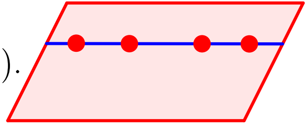

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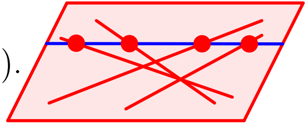

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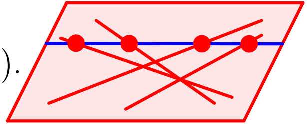

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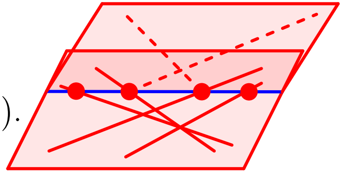

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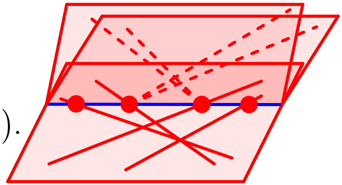

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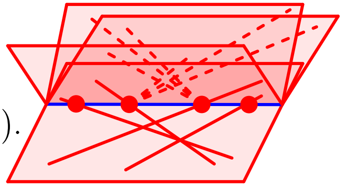

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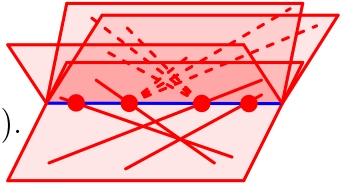

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To simplify things, we consider a code word $\boldsymbol{c} \in \boldsymbol{C_2(3, p)}$, with

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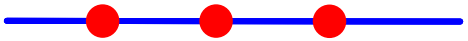

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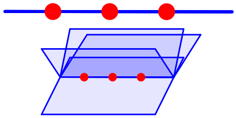

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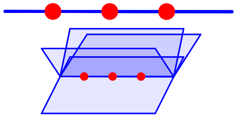

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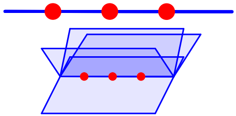

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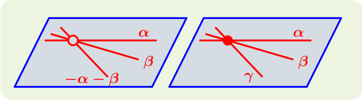

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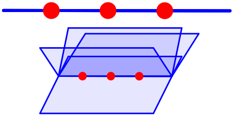

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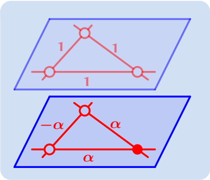

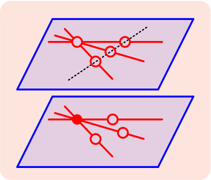

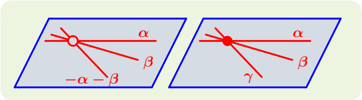

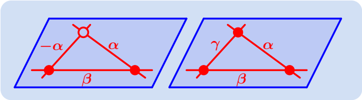

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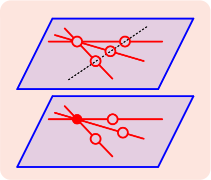

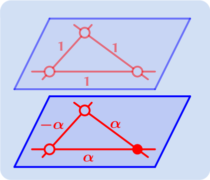

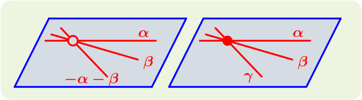

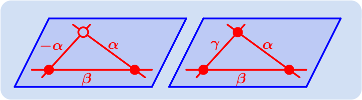

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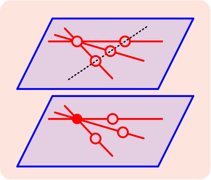

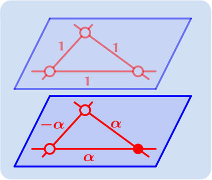

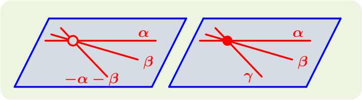

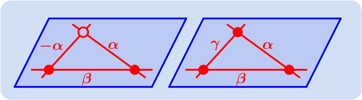

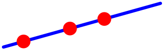

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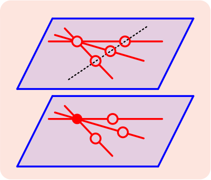

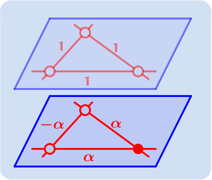

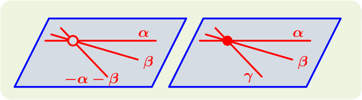

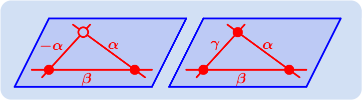

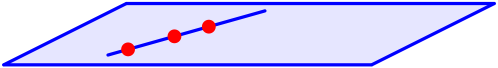

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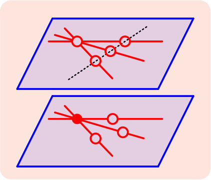

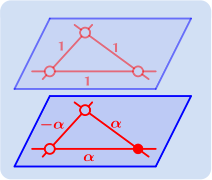

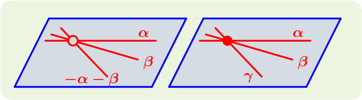

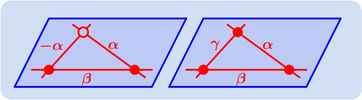

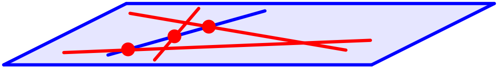

Part 2 of proof: stitching planes together

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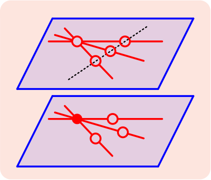

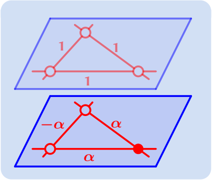

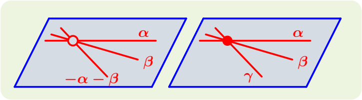

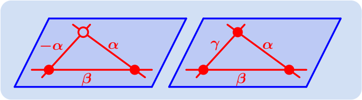

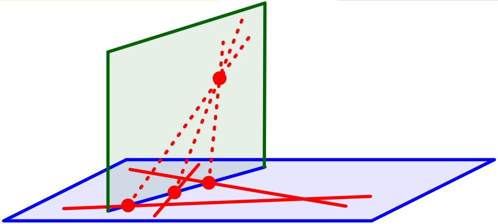

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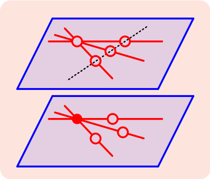

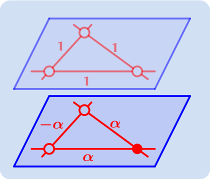

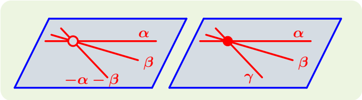

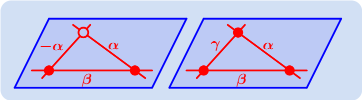

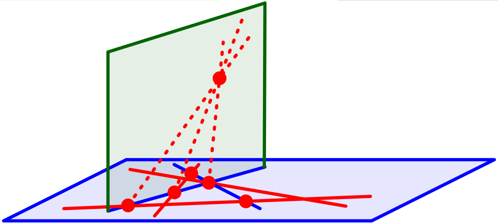

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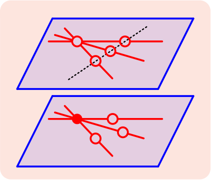

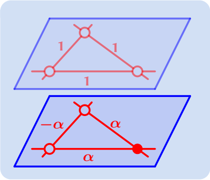

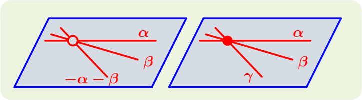

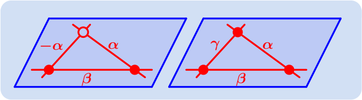

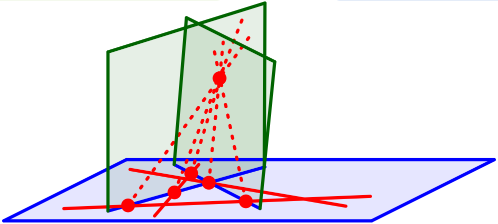

8 Results & further research

Our result: all small code words are cones

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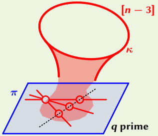

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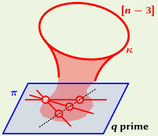

arXiv:1905.04978

Results & further research

Szőnyi & Weiner: the plane ($q = p^h$, $h \geqslant 2$, $q > 27$)

Code words of weight lower than $\frac{(p-1)(p-4)(p^2+1)}{2p-1}$, when $h = 2$,

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Our result: further classification ($q = p^h$, $h \geqslant 2$, $q > 27$)

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8 Results & further research

The code of j- and k-spaces

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'An Investigation into Small Weight Code Words
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Sam Adriaensen
Today - 13:50

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Today - 13:50 - ... like now

Fin.

Thank you for your attention. Are there any questions?

9 Questions & answers

Fin.ite geometry is awesome!

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