A problem of particular interest in recent years is to decide which families of finite (almost) simple groups arise as automorphism groups of certain highly symmetric combinatorial structures known as abstract regular polytopes. For a given group, the question may be formulated in terms of the existence of generating tuples of involutions satisfying certain stringent conditions. This talk will survey known polytope constructions for groups of Lie type and present new constructions for classical groups of low Lie rank defined over fields of characteristic 2