Algebraic techniques in finite geometry: a case study

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Consder the affine plane $\mathrm{AG}(2, q)$. Given any set of points A, an element $m \in \mathrm{GF}(q)$ is called *a direction determined by* A if m is the slope of any line meeting at least two points of A. Typical problems are to determine how many directions certain pointsets determine, or to determine the geometrical structure of the pointset A when the number of directions is given.

These problems have been studied by several people using *polynomial techniques*. Essentially, a fully reducible polynomial f is associated to the pointset A, properties of the polynomial can be deduced algebraically, and a lot of results of [1] then help to determine the polynomial completely. This gives a geometrical characterization of the pointset A.

In this talk, we will illustrate this process by proving the non-existence of a maximal partial ovoid of the generalized quadrangle $T_2(\mathcal{O})$ of size $q^2 - 1$ for certain values of q. We will introduce the point-line geometry $T_2(\mathcal{O})$, define the concept *maximal partial ovoid*, explain some combinatorial properties of a maximal partial ovoid of size $q^2 - 1$ and then show that this problem is actually a direction problem in the affine space $\mathrm{AG}(3, q)$. We illustrate the association of the Rédei-polynomial to a pointset related to the object, and prove the non-existence by analyzing algebraic properties of the polynomial. We will end the talk by discussing remaining difficulties.

References

[1] L. Rédei. *Lacunary polynomials over finite fields.* North-Holland Publishing Co., Amsterdam, 1973. Translated from the German by I. Földes.