On the (weak) cylinder conjecture

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The cylinder conjecture is the following statement.

> Let q be prime, and let U be a set of q^2 points in the affine space $\mathrm{AG}(3, q)$ such that every plane of $\mathrm{AG}(3, q)$ meets U in $0 \pmod q$ points. Then U is the set of points of a cylinder, i.e. the set of points on q lines of the same parallel class.

With a combinatorial approach it is possible to show this conjecture for very small q, i.e. $q \in \{3, 5\}$. We discuss the difficulties for higher values of q. An alternative approach is based on the use of Rédei-polynomials. However, it seems that this approach is only has some extra power when assuming the extra condition that U does not determine at least q directions. Based on this extra assumption, we discuss the algebraic conditions that can be derived, and which lead, using a computational approach, to a proof of the weaker conjecture for slightly larger values of q, i.e. $q \in \{11, 13\}$.