Direction problems in affine spaces, related problems, and applications

Jan De Beule

Ghent University

Let $\mathrm{AG}(n,q)$ denote the n-dimensional affine space over the finite field $\mathbb{F}_q$. Consider an arbitrary set of points $U \subset \mathrm{AG}(n,q)$. A point at infinity P is called a *direction determined by* U if and only if there exist two points in U of determining an affine line containing P as point at infinity. Clearly all points at infinity are directions determined by U if $|U| > q^{n-1}$. Denote by U_D the set of directions determined by U. The following research questions are of our interest.

- What are the possible sizes of U_D given that $|U| = q^{n-1}$? What is the possible structure of U_D?
- What are the possible sets U, $|U| = q^{n-1}$, given that U_D (or its complement at infinity) or only $|U_D|$ is known?
- Given that a set N of directions is not determined by a set U, $|U| = q^{n-1} - \epsilon$, can U be extended to a set U', $|U'| = q^{n-1}$, such that U' does not determine the given set N?

Many results are known for $n = 2$. We focus on particular results, their application in the theory of blocking sets of finite Desarguesian projective planes, and the use of so-called *lacunary polynomials* to obtain results. Then we describe more recent results for $n \geq 3$, of which some of them are joint work with A. Gács, P. Sziklai and M. Takáts. Finally, we present connections between direction problems in $\mathrm{AG}(3,q)$ and the *Cylinder conjecture*.

Ghent University, Dept. Mathematics, Krijgslaan 281, S22, B-9000 Gent, Belgium
jdebeule@cage.ugent.be