

Abstract

On maximal partial spreads of the hermitian variety

$H(3, q^2)$

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We consider the hermitian variety $H(3, q^2)$ as the geometry consisting of all totally isotropic subspaces with respect to a given non-singular hermitian form on the projective space $PG(3, q^2)$. It consists of points and lines, and it is one of the finite classical generalized quadrangles.

A *spread* is a set \mathcal{L} of lines of $H(3, q^2)$ partitioning the point set of $H(3, q^2)$. It is known for a long time that no spreads of $H(3, q^2)$ exist. A *partial spread* is a set \mathcal{L} of lines of $H(3, q^2)$ such that every point of $H(3, q^2)$ is contained in at most one element of \mathcal{L} . A partial spread is called *maximal* if it is not a proper subset of any (partial) spread. The natural question is how large a maximal partial spread of $H(3, q^2)$ can be.

We discuss the currently best known upper bound on the size of maximal partial spreads of $H(3, q^2)$. This upper bound is sharp for $q = 2$ and $q = 3$, but probably not for all $q > 3$. Computer searches confirm this for $q = 4$ and $q = 5$. We discuss known examples of *large* maximal partial spreads of $H(3, q^2)$.