ELEC system identification workshop

Exercises

Ivan Markovsky

Outline

Line fitting

problem: fit points $d_1, \ldots, d_N \in \mathbb{R}^2$ by a line

1. find condition for existence of a line (any line in $\mathbb{R}^2$) that passes through the points

2. how would you test the condition in MATLAB?

3. implement a method for exact line fitting

Conic section fitting

problem: fit points $d_1, \dots, d_N \in \mathbb{R}^2$ by conic section

$$\mathscr{B}(S,u,v) = \{\, d \in \mathbb{R}^2 \mid d^\top S d + u^\top d + v = 0 \,\}$$

1. find condition for existence of an exact fit
2. propose numerical method for exact fitting
3. implement the method and test it on the data

$$d_1 = \begin{bmatrix} -1 \\ -1 \end{bmatrix}, \quad d_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad d_3 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad d_4 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

Recursive sequence fitting

problem: fit $w = \big(w(1), \dots, w(T)\big)$ by model

$$\mathscr{B} = \{\, w \mid R_0 w + R_1 \sigma w + \cdots + R_\ell \sigma^\ell w = 0 \,\}$$

1. find condition for existence of an exact fit
   first, with, and then, without knowledge of ℓ

2. propose numerical method for exact fitting
   find the smallest ℓ, for which exact model exists

3. implement the method and test it on the data

$$(1, 2, 4, 7, 13, 24, 44, 81)$$

Outline

Checking whether a sequence is trajectory

1. given sequence w and polynomial R, propose method for checking numerically whether $w \in \mathscr{B} = \ker\big(R(\sigma)\big)$

2. implement it in a function `w_in_ker(w, r)`

3. test it on the trajectory

$$w = (u_d, y_d) = \big((0,1),(0,1),(0,1),(0,1)\big)$$

   and the system

$$\mathscr{B} = \ker\big(R(\sigma)\big), \qquad R(z) = \begin{bmatrix} 1 & -1 \end{bmatrix} + \begin{bmatrix} -1 & 1 \end{bmatrix} z$$

Transfer function $\mapsto$ kernel representation

1. what model $\mathscr{B}_{\text{tf}}(H)$ is specified by transfer function

$$H(z) = \frac{q(z)}{p(z)} = \frac{q_0 + q_1 z^1 + \cdots + q_\ell z^\ell}{p_0 + p_1 z^1 + \cdots + p_\ell z^\ell}$$

2. find R, such that

$$\mathscr{B}_{\text{tf}}(H) = \ker(R)$$

3. write function `tf2r` converting H (`tf` object) to R
 and function `r2tf` converting R to H

Initial conditions specification by trajectory

```
LSIM(SYS,U,T,X0) specifies the initial
state vector X0 at time T(1)
  (for state-space models only).
```

problem: given minimal $\mathscr{B} = \mathscr{B}(A, B, C, D) \in \mathscr{L}_{\mathrm{m},\ell}$

1. show that $\underbrace{\big(w(-\ell+1), \dots, w(0)\big)}_{w_\mathrm{p}} \in \mathscr{B}$ determines $x(0)$

2. explain how to use w_p to "set" given $x(0)$

3. implement and test $w_\mathrm{p} \leftrightarrow x(0)$ (`wp2x0` / `x02wp`)

Outline

Exact identification of a kernel representation

let $w \in \mathscr{B} \in \mathscr{L}^2_{1,\ell}$ (SISO system)

implement the method $w \mapsto R$ (slide 19)

test it on examples (use `drss`)

Impulse response estimation

let $w \in \mathscr{B} \in \mathscr{L}^2_{1,\ell}$ (SISO system)

implement the method $w \mapsto H$ (slide 20–21)

test it on examples (use `drss`)

Outline

Misfit computation using image repr.

given

- data $w = \big(w(1), \dots, w(T)\big)$ and
- LTI system $\mathscr{B} = \text{image}\big(P(\sigma)\big)$

derive method for computing

$$\text{misfit}(w, \mathscr{B}) := \min_{\widehat{w} \in \mathscr{B}} \|w - \widehat{w}\|_2$$

i.e., find the orthogonal projection of w on $\mathscr{B}$

Misfit computation using I/S/O representation

given

- data $w = \big(w(1), \dots, w(T)\big)$ and
- LTI system $\mathscr{B} = \mathscr{B}(A, B, C, D)$

derive method for computing

$$\operatorname{misfit}(w, \mathscr{B}) := \min_{\widehat{w} \in \mathscr{B}} \|w - \widehat{w}\|_2$$

i.e., find the orthogonal projection of w on $\mathscr{B}$

Latency computation using kernel repr.

given

- data w and
- LTI system $\mathscr{B}_{\text{ext}} = \ker\big(R(\sigma)\big)$ $\qquad (w_{\text{ext}} := \left[\begin{smallmatrix}\widehat{e}\\ w\end{smallmatrix}\right])$

find an algorithm for computing

$$\text{minimize} \quad \text{over } e \quad \|\widehat{e}\| \quad \text{subject to} \quad (\widehat{e}, w) \in \mathscr{B}_{\text{ext}}$$

Outline

Software

mosaic-Hankel low-rank approximation

http://slra.github.io/software.html

```
[sysh,info,wh] = ident(w, m, ell, opt)
```

- `sysh` — I/S/O representation of the identified model
- `opt.sys0` — I/S/O repr. of initial approximation
- `opt.wini` — initial conditions
- `opt.exct` — exact variables
- `info.Rh` — parameter R of kernel repr.
- `info.M` — misfit

```
[M, wh, xini] = misfit(w, sysh, opt)
```

demo file

Variable permutation

verify that permutation of the variables doesn't change the optimal misfit

```
T = 100; n = 2; B0 = drss(n);
u = randn(T, 1); y = lsim(B0, u) + 0.001 * ranc
[B1, info1] = ident([u y], 1, n); disp(info1.M)
   2.9736e-05
[B2, info2] = ident([y u], 1, n); disp(info2.M)
   2.9736e-05
disp(norm(B1 - inv(B2)))
   5.8438e-12
```

Output error identification

verify that the results of `oe` and `ident` coincide

```
T = 100; n = 2; B0 = drss(n);
u = randn(T, 1); y = lsim(B0, u) + 0.001 * ranc
opt = oeOptions('InitialCondition', 'estimate')
B1 = oe(iddata(y, u), [n + 1 n 0], opt);
B2 = ident([u y], 1, n, struct('exct', 1));
norm(B1 - B2) / norm(B1)

ans =

   1.4760e-07
```