Scale Relativity

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Clément Vidal
contact@clemvidal.com
http://www.clemvidal.com

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0 - Introduction

Scale relativity is a geometrical and fractal space-time theory. The idea of a fractal space-time theory has been first introduced by Garnet Ord (1983), and by Laurent Nottale in a paper with Jean Schneider (1984). The proposal to combine fractal space-time theory with relativity principles was made by Laurent Nottale (Nottale 1989; 1992). The resulting scale relativity theory is an extension of the concept of relativity found in special relativity and general relativity to physical scales (time, length, energy, or momentum scales). In physics, relativity theories have shown that position, orientation, movement and acceleration can't be defined in an absolute way, but only relatively to a system of reference.

Noticing the relativity of scales, as noticing the other forms of relativity is just a first step. Scale relativity theory proposes to make the next step by translating this simple insight formally in physical theory, by introducing explicitly in coordinate systems the "state of scale".

To describe scale transformations requires the use of fractal geometries, which are typically concerned with scale changes. Scale relativity is thus an extension of relativity theory to the concept of scale, using fractal geometries to study scale transformations.

The construction of the theory is similar to previous relativity theories, with three different levels: galilean, special and general. The development of a full general scale relativity is not finished yet. However, the existing progress and results already have consequences for the foundations of quantum mechanics, particle physics, and high energy physics. Furthermore, empirical predictions in physics, astrophysics, and cosmology have already been validated, most often with a high precision, or highly statistically significant results.

1 - History

1.1 Feynman's paths in quantum mechanics

Richard Feynman developed a path integral formulation of quantum mechanics (Feynman and Hibbs 1965), contrasting with the Copenhagen interpretation where the notion of trajectory in space-time was abandoned. Searching for the most important paths relevant for quantum particles, Feynman noticed that such paths were very irregular on small scales, i.e. infinite and non-differentiable. This means that in between two points, a particle can have not one path, but an infinity of potential paths.

This can be illustrated with a concrete example. Imagine that you are hiking in the mountains, and that you are free to walk wherever you like. To go from point A to point B, there is not one shortest path, but an infinity of possible paths, each going through different valleys and hills (Müller 2005, 71).

Scale relativity hypothesizes that quantum behavior comes from the fractal nature of spacetime. Indeed, fractal geometries allow to study such non-differentiable paths. This fractal interpretation of quantum mechanics has been further specified by Abbot and Wise (1981), showing that the paths have a fractal dimension 2. Scale relativity goes one step further by asserting that the fractality of these paths is a consequence of the fractality of space-time.

There are other pioneers who saw the fractal nature of quantum mechanical paths (Campesino-Romeo, D'Olivo, and Socolovsky 1982; Allen 1983). Also, as much as the development of general relativity required the mathematical tools of non-Euclidean (Riemanian) geometries, the development of a fractal space-time theory would not have been possible without the concept of fractal geometries developed and popularized by Benoît Mandelbrot. Fractals are usually associated with the self-similar case of a fractal curve, but other more complicated fractals are possible, e.g. considering not only curves, but also fractal surfaces or fractal volumes, as well as investigating fractal dimensions which have other values than 2, and which also vary with scale.

1.2 Independent discovery

Garnet Ord (1983) and Laurent Nottale (in Nottale and Schneider 1984) both connected fractal space-time with quantum mechanics. Nottale coined the term "scale relativity" in 1992 (Nottale 1992). He developed the theory and its applications with more than one hundred scientific papers (see here), two technical books in English (Nottale 1993a; Nottale 2011), and three popular books in French (Nottale 1998a; Nottale, Chaline, and Grou 2000; 2009).

2 - Basic concepts

2.1 Principle of scale relativity

The principle of relativity says that physical laws should be valid in all coordinate systems. This principle has been applied to states of position (the origin and orientation of axes), as well as to the states of movement of coordinate systems (speed, acceleration). Such states are never defined in an absolute manner, but relatively to one another. For example, there is no absolute movement, in the sense that it can only be defined in a relative way between one body and another. Scale relativity proposes in a similar manner to define a scale relative to another one, and not in an absolute way. Only scale ratios have a physical meaning, never an absolute scale, in the same way as there exists no absolute position or velocity, but only position or velocity differences.

The concept of *resolution* is re-interpreted as the "state of scale" of the system, in the same way as velocity characterizes the state of movement. The principle of scale relativity can thus be formulated as:

> "the laws of physics must be such that they apply to coordinate systems whatever their state of scale."
> (Nottale 2011, 8)

The main goal of scale relativity is to find laws which mathematically respect this new principle of relativity. Mathematically, this can be expressed through the principle of covariance applied to scales, that is, the invariance of the form of physics equations under transformations of resolutions (dilations and contractions).

2.2 Including resolutions in coordinate systems

Galileo introduced explicitly velocity parameters in the observational referential. Then, Einstein introduced explicitly acceleration parameters. In a similar way, Nottale introduces scale parameters explicitly in the observational referential. The core idea of scale-relativity is thus to include resolutions explicitly in coordinate systems, thereby integrating measure theory explicitly in the formulation of physical laws.

An important consequence is that coordinates are not numbers anymore, but *functions*, which depend on the resolution (Nottale 1998a, 218). For example, the length of the Brittany coast is explicitly dependent on the resolution at which one measures it (Mandelbrot 1983).

If we measure a pen with a ruler graduated at a millimetric scale, we should write that it is 15 ± 0.1 cm. The error bar indicates the resolution of our measure. If we had measured the pen at another resolution, for example with a ruler graduated at the centimeter scale, we would have found another result, 15 ± 1 cm. In scale relativity, this resolution defines the "state of scale". In the relativity of movement, this is similar to the concept of speed, which defines the "state of movement".

The relative state of scale is fundamental to know about for any physical description. For example, if we want to describe the movement and properties of a sphere, we may as well use classical mechanics or quantum mechanics depending on the size of the sphere in question (Nottale 2004a).

In particular, information on resolution is essential to understand quantum mechanical systems, and in scale relativity, resolutions are included in coordinate systems, so it seems a logical and promising approach to account for quantum phenomena.

2.3 Dropping the hypothesis of differentiability

Scientific theories usually do not improve by adding complexity, but rather by starting from a more and more simple basis. This fact can be observed throughout the history of science. The reason is that starting from a less constrained basis provides more freedom and therefore allows richer phenomena to be included in the scope of the theory. Therefore, new theories usually do not contradict the old ones, but widen their domain of validity and include previous knowledge as special cases. For example, releasing the constraint of rigidity of space led Einstein to derive his theory of general relativity and to understand gravitation. As expected, this theory naturally includes Newton's theory, which is recovered as a linear approximation under weak fields.

The same type of approach has been followed by Nottale to build the theory of scale relativity. The basis of current theories is a continuous and two-times differentiable space. Space is by definition a continuum, but the assumption of differentiability is not supported by any fundamental reason. It is usually assumed only because it is observed that the first two derivatives of position with respect to time are needed to describe motion. Scale relativity theory is rooted in the idea that the constraint of differentiability can be relaxed and that this allows quantum laws to be derived.

In terms of geometry, differentiability means that a curve is sufficiently smooth and can be approximated by a tangent. Mathematically, two points are placed on this curve and one observes the slope of the straight line joining them as they become closer and closer. If the curve is smooth enough, this process converges (almost) everywhere and the curve is said to be differentiable. It is often believed that this property is common in nature. However, most natural objects have instead a very rough surface, or contour. For example the bark of trees and snowflakes have a detailed structure that does not become smoother when the scale is refined. For such curves, the slope of the tangent fluctuates endlessly or diverges. The derivative is then undefined (almost) everywhere and the curve is said to be nondifferentiable.

Therefore, when the *assumption* of space differentiability is abandoned, there is an additional degree of freedom that allows the geometry of space to be extremely rough. The difficulty in this approach is that new mathematical tools are needed to model this geometry because the classical derivative cannot be used. Nottale found a solution to this problem by using the fact that nondifferentiability implies scale dependence and therefore the use of fractal geometry. Scale dependence means that the distances on a nondifferentiable curve depend on the scale of observation. It is therefore possible to maintain differential calculus provided that the scale at which derivatives are calculated is given, and that their definition includes no limit. It amounts to saying that nondifferentiable curves have a whole set of tangents in one point instead of one, and that there is a specific tangent at each scale.

To abandon the hypothesis of differentiability does not mean abandoning differentiability. Instead, this leads to a more general framework, where *both* differentiable and non-differentiable cases are included. Combined with motion relativity, scale relativity by definition thus extends and contains general relativity.

As much as general relativity is possible when we drop the hypothesis of euclidian space-time, allowing the possibility of curved space-time, scale relativity is possible when we abandon the hypothesis of differentiability, allowing the possibility of a fractal space-time. The objective is then to describe a continuous space-time which is not everywhere differentiable, as it was in general relativity.

Abandoning differentiability doesn't mean abandoning differential equations. The concept of fractal allows to work with the nondifferentiable case with differential equations.

In differential calculus, we can see the concept of limit as a zoom, but in this generalization of differential calculus, one doesn't look anymore only at the limit zooms (zero and infinity) but also everything in between, that is, all possible zooms.

In sum, we can drop the hypothesis of the differentiability of space-time, keeping differential equations, provided that fractal geometries are used. With them, we can still deal with the nondifferentiable case with the tools of differential equations. This leads to a double differential equation treatment: in space-time and in scale space.

2.4 Fractal space-time

If Einstein showed that space-time was curved, Nottale shows that it is not only curved, but also fractal. Nottale (1993a, 82) has proven a key theorem which shows that a space which is continuous and non-differentiable is necessarily fractal. It means that such a space depends on scale.

Importantly, the theory does not merely describes fractal objects in a given space. Instead, it is *space itself which is fractal*. To understand what a fractal space means requires to study not just fractal curves, but also fractal surfaces, fractal volumes, etc.

Mathematically, a fractal space-time is defined as a nondifferentiable generalization of Riemanian geometry (Nottale 1993a, 84; Nottale 1998a, 188). Such a fractal space-time geometry is the natural choice to develop this new principle of relativity, in the same way that curved geometries were needed to develop Einstein's theory of general relativity (Nottale 2013a).

In the same way that general relativistic effects are not felt in a typical human life, the most radical effects of the fractality of spacetime appear only at the extreme limits of scales: micro scales or at cosmological scales. This approach therefore proposes to bridge not only the quantum and the classical, but also the classical and the cosmological, with fractal to non-fractal transitions (see Fig. 1).

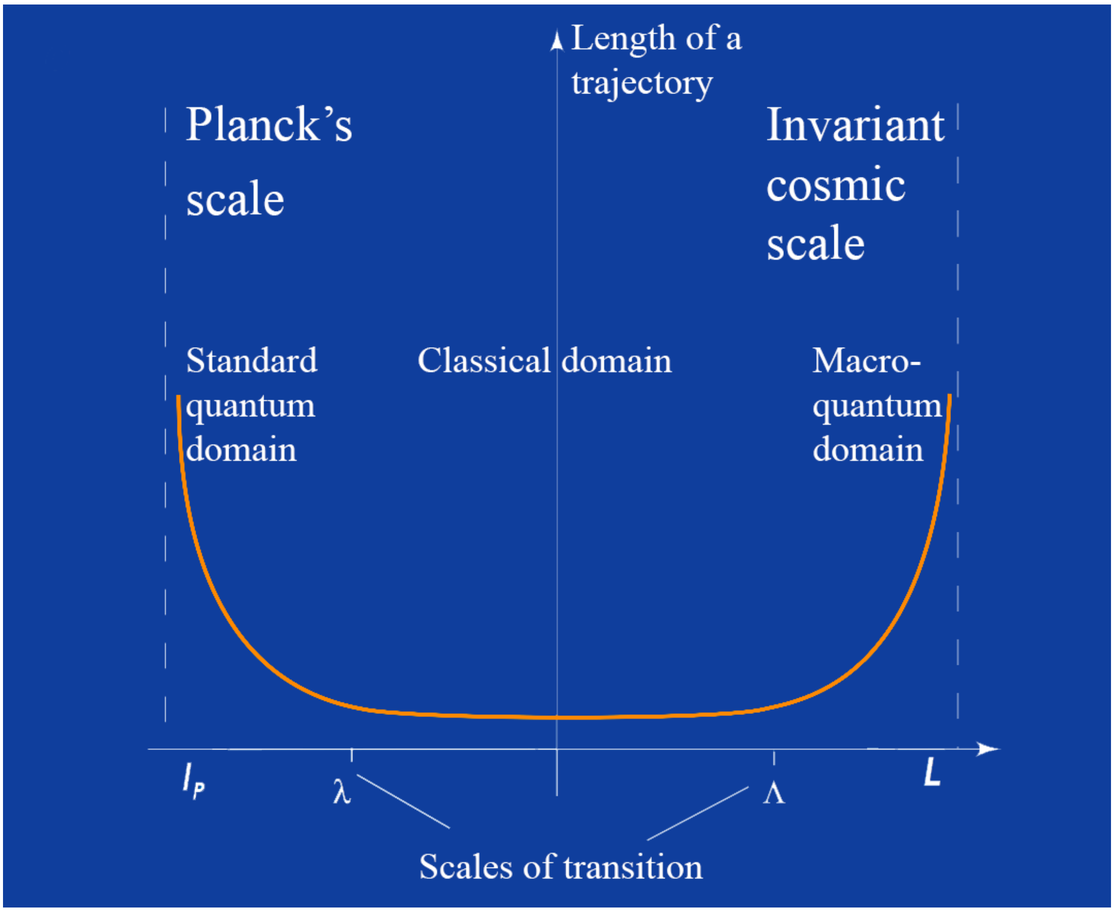

Fig. 1. Variation of the fractal dimension of space-time geodesics (trajectories), according to the resolution, in the framework of special scale relativity. The scale symmetry is broken in two transitioning scales λ and Λ (non-absolute), which divide the scale space in three domains: (1) a classical domain, intermediary, where space-time doesn't depend on resolutions because the laws of movement dominate over scale laws; and two asymptotical domains towards (2) very small and (3) very large scales where scale laws dominate over the laws of movement, which makes explicit the fractal structure of space-time. For more plots about these transitions, see (Nottale 2003a; 1993a, 304; 1996a, 915).

2.5 Minimum and maximum invariant scales

A fundamental and elegant result of scale relativity is to propose a minimum and maximum scale in physics, invariant under dilations, in a very similar way as the speed of light is an upper limit for speed.

2.5.1 Minimum invariant scale

In special relativity, there is an unreachable speed, the speed of light. We can add speeds without end, but they will always be less than the speed of light. The sums of all speeds are limited by the speed of light. Additionally, the composition of two velocities is inferior to the sum of those two speeds.

In special scale relativity, similar unreachable observational scales are proposed, the Planck length scale (l_P) and the Planck time scale (t_P). Dilations are borned by l_P and t_P, which means that we can divide spatial or temporal intervals without end, but they will always be superior to Planck's length and time scales. This is a result of special scale relativity (see section 2.7 below). Similarly, the composition of two scale changes is inferior to the product of these two scales (Nottale 1998a, 161).

2.5.2 Maximum invariant scale

The choice of the maximum scale is less easy to explain, but it mostly consists to identify it with the cosmological constant: $L = 1/(\Lambda^2)$ (Nottale 1993a, 299; Nottale 1996a; Nottale 2003b; Nottale 2011, chap. 12.6). This is motivated in parts because a dimensional analysis shows that the cosmological constant is the inverse of the square of a length, i.e. a curvature.

2.6 Galilean scale relativity

The theory of scale relativity follows a similar construction as the one of the relativity of movement, which took place in three steps: galilean, special and general relativity.

This is not surprising, as in both cases the goal is to find laws satisfying transformation laws including one parameter that is relative: the speed in the case of the relativity of movement; the resolution in the case of the relativity of scales.

Galilean scale relativity involves linear transformations a constant fractal dimension, self-similarity and scale invariance. This situation is best illustrated with self-similar fractals. Here, the length of geodesics varies constantly with resolution. The fractal dimensions of free particles doesn't change with zooms. These are self-similar curves.

In galilean relativity, recall that the laws of motion are the same in all inertial frames. Galileo (1991) famously concluded that "the movement is like nothing". In the case of self-similar fractals, paraphrasing Galileo, one could say that "scaling is like nothing". Indeed, the same patterns occur at different scales, so scaling is not noticeable, it is like nothing.

In the relativity of movement, Galileo's theory is an additive galilean group:

$$X' = X - VT$$
$$T' = T$$

However, if we consider scale transformations (dilations and contractions), the laws are products, and not sums. This can be seen by the necessity to use units of measurements. Indeed, when we say that an object measures 10 meters, we actually mean the object measures 10 **times** the definite predetermined length called "meter". The number 10 is actually a scale ratio of two lengths 10/1m, where 10 is the measured quantity, and 1m is the arbitrary defining unit. This is the reason why the group is multiplicative.

Moreover, an arbitrary scale *e* doesn't have any physical meaning in itself (like the number 10), only scale ratios $r=e'/e$ have a meaning, in our example, $r=10/1$. Using the Gell-Mann-Lévy method (Gell-Mann and Lévy 1960), we can use a more relevant scale variable, $V=\ln(e'/e)$, and then find back an additive group for scale transformations by taking the logarithm –which converts products into sums.

Interestingly, when, in addition to the principle of scale relativity, one adds the principle of relativity of movement, there is a transition of the structure of geodesics at large scales, where trajectories do not depend on the resolution anymore, where trajectories become classical. This explains the shift of behavior from quantum to classical (Nottale 1993a; 1996a). See also Fig. 1.

2.7 Special scale relativity

Special scale relativity can be seen as a correction of galilean scale relativity, where Galilean transformations are replaced by Lorentz transformations (Nottale 1992; 1993a). Interestingly, the "corrections remain small at "large" scale (i.e. around the Compton scale of particles) and increase when going to smaller length scales (i.e. large energies) in the same way as motion-relativistic corrections increase when going to large speeds." (Nottale 2011, 460).

In Galilean relativity, it was considered "obvious" that we could add speeds without limit ($w = u + v$). This composition laws for speed was not challenged. However, Poincaré and Einstein did challenge it with special relativity, setting a maximum speed on movement, the speed of light. Formally, if v is a velocity, $v + c = c$. The status of the speed of light in special relativity is a horizon, unreachable, impassable, invariant under changes of movement.

Regarding scale, we are still within a Galilean kind of thinking. Indeed, we assume without justification that the composition of two dilations is ρ x $\rho = \rho^2$. Written with logarithms, this equality becomes $ln\rho + ln\rho = 2ln\rho$. However, nothing guarantees that this law should hold at quantum or cosmic scales (Nottale 1998a, 212). As a matter of fact, this dilation law is corrected in special scale relativity, and becomes: $\ln\rho + \ln\rho = 2 \ln \rho / (1 + \ln \rho^2)$.

More generally, in special relativity the composition law for velocities differs from the galilean approximation and becomes (with the speed of light c=1):

$$u \oplus v = (u + v) / (1 + u*v)$$

Similarly, in special scale relativity, the composition law for dilations differs from our galilean intuitions and becomes (in a logarithm of base K which includes a possible constant C = ln K, which plays the same role as c):

$$\log\rho1 \oplus \log\rho2 = (\log\rho1 + \log\rho2) / (1 + \log\rho1 * \log\rho2)$$

The status of the Planck scale in special scale relativity plays a similar role as the speed of light in special relativity. It is a horizon for small scales, unreachable, impassable, invariant under scale changes, i.e. dilations and contractions. The consequence for special scale relativity is that applying two times the same contraction ρ to an object, the result is a contraction less strong than contraction ρ x ρ. Formally, if ρ is a contraction, ρ x $l_P = l_P$.

As noted above, there is also an unreachable, impassable maximum scale, invariant under scale changes, which is the cosmic length L (Nottale 1993a). In particular, it is invariant under the expansion of the universe.

2.8 General scale relativity

In galilean scale relativity, spacetime was fractal with *constant* fractal dimensions. In special scale relativity, fractal dimensions can vary. This varying fractal dimension remains however constrained by a log-Lorentz law. This means that the laws satisfy a logarithmic version of the Lorentz transformation. The varying fractal dimension is covariant, in a similar way as proper time is covariant in special relativity.

In general scale relativity, the fractal dimension is not constrained anymore, and can take any value. In other words, it is the situation where there is *curvature in scale space*. Einstein's curved space-time becomes a particular case of the more general fractal spacetime.

General scale relativity is much more complicated, technical, and less developed than its galilean and special versions. It involves non-linear laws, scale dynamics and gauge fields. In the case of non self-similarity, changing scales generates a new scale-force or scale-field which needs to be taken into account in a scale dynamics approach. Quantum mechanics then needs to be analyzed in scale space (Nottale 1997; Nottale 2004b).

Finally, in general scale relativity, we need to take into account both movement *and* scale transformations, where scale variables depend on space-time coordinates. More details about the implications for abelian gauge fields (Nottale 1994; 1996a) and non-abelian gauge fields (Nottale, Célérier, and Lehner 2006) can be found in the literature. Nottale's (2011) book provides the state of the art.

To sum up, one can see some structural similarities between the relativity of movement and the relativity of scales in Table 1.

Relativity	**Variables defining the coordinate system**	**Variables characterizing the state of the coordinate system**
Movement	Space	Speed
	Time	Acceleration
Scale	Length of a fractal	Resolution
	Variable fractal dimension	Scale acceleration

Table 1. Comparison between relativity of movement and relativity of scales.
In both cases, there are two kinds of variables linked to the coordinate systems: variables which define the coordinate system, and variables that characterize the state of the coordinate system. In this analogy, the resolution can be assimilated to a speed; acceleration to a scale acceleration; space to the length of a fractal; and time, to the variable fractal dimension (Nottale 1992). Table adapted from (Forriez, Martin, and Nottale 2010).

3 - Consequences for quantum mechanics

3.1 Introduction

The fractality of space-time implies an infinity of virtual geodesics. This remark already means that a fluid mechanics is needed. Note that this view is not new, as many authors have noticed fractal properties at quantum scales, thereby suggesting that typical quantum mechanical paths are fractal. See (Kröger 1997) for a review. However, the idea to consider a fluid of geodesics in a fractal spacetime is an original proposal from Nottale.

In scale relativity, quantum mechanical effects appear as effects of fractal structures on the movement. The fundamental indeterminism and nonlocality of quantum mechanics are deduced from the fractal geometry itself.

There is an analogy between the interpretation of gravitation in general relativity and quantum effects in scale relativity. Indeed, if gravitation is a manifestation of space-time curvature in general relativity, quantum effects are manifestations of a fractal space-time in scale relativity.

To sum up, there are two aspects which allows scale relativity to better understand quantum mechanics. On the one side, fractal fluctuations themselves are hypothesized to lead to quantum effects. On the other side, non-differentiability leads to a local irreversibility of the dynamics and therefore to the use of complex numbers.

Quantum mechanics thus receives not only a new interpretation, but a firm foundation in relativity principles.

3.2 Quantum-classical transition

As Turner (2013) summarized:

> "the structure of space has both a smooth (differentiable) component at the macro-scale and a chaotic, fractal (non-differentiable) component at the micro-scale, the transition taking place at the de Broglie length scale."

This transition is explained with galilean scale relativity (Célérier and Nottale 2004) (see also above).

3.3 Derivation of quantum mechanics' postulates

Starting from scale relativity, it is possible to derive the fundamental "postulates" of quantum mechanics (Nottale and Célérier 2007). More specifically, building on the result of the key theorem showing that a space which is continuous and non-differentiable is necessarily fractal (see section 2.4), Schrödinger's equation, Born's and von Neumann's postulate are derived.

To derive Schrödinger's equation, Nottale (1993a, sec. 5.6) started with Newton's second law of motion, and used the result of the key theorem. Many subsequent works then confirmed the derivation (e.g. Dubois 2000; Jumarie 2001; Cresson 2003; Ben Adda and Cresson 2004; 2005; Jumarie 2006; 2007).

Actually, the Schrödinger equation derived becomes generalized in scale relativity, and opens the way to a macroscopic quantum mechanics (see below for validated empirical predictions in astrophysics). This may also help to better understand macroscopic quantum phenomena in the future.

Reasoning about fractal geodesics and non-differentiability, it is also possible to derive von Neumann's postulate (Nottale 2011, sec. 5.7.2) and Born's postulate (Nottale 2011, sec. 5.7.3).

With the hypothesis of a fractal space-time, the Klein-Gordon, and the Dirac equation can then be derived (Célérier and Nottale 2003; 2010; Nottale and Célérier 2007).

The significance of these fundamental results is immense, as the foundations of quantum mechanics which were up to now axiomatic, are now logically derived from more primary relativity theory principles and methods.

3.4 Gauge transformations

Gauge fields appear when scale and movements are combined. Scale relativity proposes a geometric theory of gauge fields. As Turner (2013) explains:

> The theory offers a new interpretation of gauge transformations and gauge fields (both Abelian and non-Abelian), which are manifestations of the fractality of space-time, in the same way that gravitation is derived from its curvature.

The relationships between fractal space-time, gauge fields and quantum mechanics are technical and advanced subject-matters elaborated in details in Nottale's latest book (Nottale 2011).

4 - Consequences for elementary particles physics

4.1 Introduction

Scale relativity gives a geometric interpretation to charges, which are now "defined as the conservative quantities that are built from the new scale symmetries" (Nottale 2011, 297). Relations between mass scales and coupling constants can be theoretically established, and some of them empirically validated. This is possible because in scale relativity, the problem of divergences in quantum field theory is resolved. Indeed, in the new framework, masses and charges become finite, even at infinite energy. In special scale relativity, the possible scale ratios become limited, constraining in a geometric way the quantization of charges. Let us compare a few theoretical predictions with their experimental measures.

4.2 Fine-structure constant

Main article: Fine-structure constant

Nottale's (2011, 490) latest theoretical prediction of the fine-structure constant at the Z_0 scale is:

$$\alpha^{-1}(m_Z) = 128.92$$

By comparison, a recent experimental measure gives (Yao and et. al. 2006):

$$\alpha^{-1}(m_Z) = 128.91 \pm 0.02$$

At low energy, the theoretical fine-structure constant prediction is (Nottale 2011, 490):

$$\alpha^{-1} = 137.01 \pm 0.035;$$

which is within the range of the experimental precision:

$$\alpha^{-1} = 137.036$$

4.3 SU (2) coupling at Z scale

Here the SU(2) coupling corresponds to rotations in a three-dimensional scale-space. The theoretical estimate of the SU (2) coupling at Z scale is (Nottale 2011, 499):

$$\alpha^{-1}{}_{2\,Z} = 29.8169 \pm 0.0002$$

While the experimental value gives (Groom et al. 2000):

$$\alpha^{-1}{}_{2\,Z} = 29.802 \pm 0.027.$$

4.4 Strong nuclear force at Z scale

Special scale relativity predicts the value of the strong nuclear force with great precision, as later experimental measurements confirmed. The first prediction of the strong nuclear force at the Z energy level was made in 1992 (Nottale 1992):

$$\alpha_S\,(m_Z) = 0.1165 \pm 0.0005$$

A recent and refined theoretical estimate gives (Nottale 2010, 123–124):

$$\alpha_S\,(m_Z) = 0.1173 \pm 0.0004,$$

which fits very well with the experimental measure (Yao et. al. 2006):

$$\alpha_S\,(m_Z) = 0.1176 \pm 0.0009$$

4.5 Mass of the electron

As an application from this new approach to gauge fields, a theoretical estimate of the mass of the electron (m_e) is possible, from the experimental value of the fine-structure constant. This leads a very good agreement (Nottale 2011, 483):

$$m_e(\text{theoretical}) = 1.007\ m_e\ (\text{experimental})$$

5 - Astrophysical applications

5.1 Macroquantum mechanics

Some chaotic systems can be analyzed thanks to a macroquantum mechanics. The main tool here is the generalized Schrödinger equation, which brings statistical predictability characteristic of quantum mechanics into other scales in nature. The equation predicts probability density peaks. For example, the position of exoplanets can be predicted in a statistical manner. The theory predicts that planets have more chances to be found at such or such distance from their star. As Baryshev and Teerikorpi (2002, 256) write:

> "With his equation for the probability density of planetary orbits around a star, Nottale has seemingly come close to the old analogy which saw a similarity between our solar system and an atom in which electrons orbit the nucleus. But now the analogy is deeper and mathematically and physically supported: it comes from the suggestion that chaotic planetary orbits on very long time scales have preferred sizes, the roots of which go to fractal space-time

and generalized Newtonian equation of motion which assumes the form of the quantum Schrödinger equation."

However, as Nottale (2011, 589) acknowledges, this general approach is not totally new:

> "The suggestion to use the formalism of quantum mechanics for the treatment of macroscopic problems, in particular for understanding structures in the solar system, dates back to the beginnings of the quantum theory"

5.2 *Solar systems*

5.2.1 Space debris

Main article: Space debris

At the scale of Earth's orbit, space debris probability peaks at 718km and 1475km have been predicted with scale relativity (da Rocha and Nottale 2003, 577), which is in agreement with observations at 850km and 1475km (Anz 2000). Da Rocha and Nottale suggest that the dynamical braking of the Earth's atmosphere may be responsible for the difference between the theoretical prediction and the observational data of the first peak.

5.2.2 Titius-Bode law

Main article: Titius-bode law

Scale relativity predicts a new law for interplanetary distances, which sheds a new light on the so-called Titius-bode "law" (Nottale 1993a, 311–321; Nottale, Schumacher, and Gay 1997). However, the predictions here are statistical and not deterministic as in Newtonian dynamics. In addition to being statistical, the scale relativistic law has a different theoretical form, and is more reliable than the original Titius-Bode version (Nottale 2011, 559):

> "The Titius-Bode "law" of planetary distance is of the form $a + b \times c^n$, with a = 0.4 AU, b = 0.3 AU and c = 2 in its original version. It is partly inconsistent — Mercury corresponds to $n = -\infty$, Venus to n = 0, the Earth to n = 1, etc. It therefore "predicts" an infinity of orbits between Mercury and Venus and fails for the main asteroid belt and beyond Saturn. It has been shown by Herrmann (1997) that its agreement with the observed distances is not statistically significant. […] [I]n the scale relativity framework, the predicted law of distance is not a Titius-Bode-like power law but a more constrained and statistically significant quadratic law of the form $a_n = a_0 n^2$."

5.2.3 Exoplanets

Main article: Exoplanet

The method also applies to other solar systems. Let us illustrate this with the first exoplanets found around the pulsar PSR B1257+12 (Wolszczan and Frail 1992).

Three planets, A, B and C have been found. Their orbital period ratios (noted P_A/P_C for the period ratio of planet A to C) can be estimated and compared to observations (Nottale 1998b). Using the macroscopic Schrödinger equation, the recent theoretical estimates (Nottale 2011, 622) are:

$$(P_A/P_C)^{1/3} = 0.63593 \text{ (predicted)}$$
$$(P_B/P_C)^{1/3} = 0.8787 \text{ (predicted)},$$

which fit the observed values (Konacki and Wolszczan 2003) with great precision:

$$(P_A/P_C)^{1/3} = 0.63597 \text{ (observed)}$$
$$(P_B/P_C)^{1/3} = 0.8783 \text{ (observed)}.$$

Interestingly, the puzzling fact that many exoplanets (e.g. hot jupiters) are so close to their parent stars receives a natural explanation in this framework. Indeed, it corresponds to the fundamental orbital of the model, where (exo)planets are at 0.04 UA / solar mass of their parent star (see Nottale 2011, sec. 13.5).

More validated predictions can be found regarding orbital periods and the distances of planets from their parent star (Nottale 1996b; 1998c; Nottale, Schumacher, and Lefevre 2000).

5.3 Galaxy pairs

Daniel da Rocha (2004) studied the velocity of about 2000 galaxy pairs, which gave statistically significant results when compared to the theoretical structuration in phase space from scale relativity. The method and tools here are similar to the one used for explaining the structure in solar systems.

Similar successful results apply at other extragalactic scales: the local group of galaxies, clusters of galaxies, the local supercluster and other very large scale structures (Nottale 2011, sec. 13.8).

5.4 Dark Matter

Main article: Dark matter

Scale relativity suggests that the fractality of matter contributes to the phenomenon of dark matter. Indeed, some of the dynamical and gravitational effects which seem to require unseen matter are suggested to be consequences of the fractality of space on very large scales (Nottale 2011, 520).

In the same way as quantum physics differs from the classical at very small scales because of fractal effects, symmetrically, at very large scales, scale relativity also predicts that corrections from the fractality of space-time must be taken into account (see also Fig. 1).

Such an interpretation is somehow similar in spirit to modified newtonian dynamics (MOND), although here the approach is founded on relativity principles. Indeed, in MOND, newtonian dynamics is modified in an *ad hoc* manner to account for the new effects, while in scale relativity, it is the new fractal geometric field taken into consideration which leads to the emergence of a dark potential.

On the largest scale, scale relativity offers a new perspective on the issue of redshift quantization. With a reasoning similar to the one which allows to predict probability peaks for the velocity of planets, this can be generalized to larger intergalactic scales. Nottale (2011, 656) writes:

> "In the same way as there are well-established structures in the position space (stars, clusters of stars, galaxies, groups of galaxies, clusters of galaxies, large scale structures), the velocity probability peaks are simply the manifestation of structuration in the velocity space. In other words, as it is already well-known in classical mechanics, a full view of the structuring can be obtained in phase space."

6 - Cosmological applications

6.1 *Large numbers hypothesis*

Main article: Dirac large numbers hypothesis

Nottale (1993a, 303) noticed that reasoning about scales was a promising road to explain the large numbers hypothesis. This was elaborated in more details in a working paper (Nottale 1993b). The scale-relativistic way to explain the large numbers hypothesis was later discussed by Nottale (e.g. Nottale 1996a; 2011, 543–545) and by (Sidharth 2001).

6.2 *Prediction of the cosmological constant*

Main article: Cosmological constant

In scale relativity, the cosmological constant is interpreted as a curvature. If one does a dimensional analysis, it is indeed the inverse of the square of a length. The predicted value of the cosmological constant, back in 1993 was (Nottale 1993a, 305):

$$\Omega_\Lambda h^2 = 0.36$$

Depending on model choices, the most recent predictions give the following range (Nottale 2012, 39):

$$0.311 < \Omega_\Lambda h^2 \text{ (predicted)} < 0.356,$$

while the measured cosmological constant from the Planck satellite (Planck Collaboration et al. 2013) is:

$$\Omega_\Lambda h^2 \text{ (measured)} = 0.318 \pm 0.012.$$

Given the improvements of the empirical measures from 1993 until 2011, Nottale (2011, 554) commented:

> "The convergence of the observational values towards the theoretical estimate, despite an improvement of the precision by a factor of more than 20, is striking."

Dark energy can be considered as a measurement of the cosmological constant. In scale relativity, dark energy would come from a potential energy manifested by the fractal geometry of the universe at large scales (Nottale 2011, 543), in the same way as the newtonian potential is a manifestation of its curved geometry in general relativity.

6.3 Horizon problem

Main article: horizon problem

Scale relativity offers a new perspective on the old horizon problem in cosmology. The problem states that different regions of the universe have not had contact with each others because of the great distances between them, but nevertheless they have the same temperature and other physical properties. This should not be possible, given that the transfer of information (or energy, heat, etc.) can occur, at most, at the speed of light.

Nottale (1993a, 292) writes that special scale relativity "naturally solves the problem because of the new behaviour it implies for light cones. Though there is no inflation in the usual sense, since the scale factor time dependence is unchanged with respect to standard cosmology, there is an inflation of the light cone as $t \to \Lambda/c$", where Λ is the Planck length scale $(\hbar G/c^3)^{1/2}$. This inflation of the light cones makes them flare and cross themselves, thereby allowing a causal connection between any two points, and solving the horizon problem (see also Nottale 2003b).

7 - Applications to other fields

Although scale relativity started as a spacetime theory, its methods and concepts can and have been used in other fields. For example, quantum-classical kinds of transitions can be at play at intermediate scales, provided that there exists a fractal medium which is locally nondifferentiable. Such a fractal medium then plays a role similar to that played by fractal spacetime for particles. Objects and particles embedded in such a medium will acquire macroquantum properties. As examples, we can mention gravitational structuring in astrophysics (see section 5), turbulence (e.g. Dubrulle 2000), supraconductivity at laboratory scales (see section 7.1), and also modeling in geography (section 7.4).

What follows are *not* strict applications of scale relativity, but rather models constructed with the general idea of relativity of scales (Nottale, Chaline, and Grou 2000; 2002). Fractal models, and in particular self-similar fractal laws have been applied to describe numerous biological systems such as trees, blood networks, or plants. It is thus to be expected that the mathematical tools developed through a fractal space-time theory can have a wider variety of applications to describe fractal systems.

7.1 Superconductivity and macroquantum phenomena

Main article: Macroscopic quantum phenomena

The generalized Schrödinger equation, under certain conditions, can apply to macroscopic scales. This leads to the proposal that quantum-like phenomena need not to be only at quantum scales. In a recent paper, Turner and Nottale (2015) proposed new ways to explore the origins of macroscopic quantum coherence in high temperature super conductivity.

7.2 Morphogenesis

If we assume that morphologies come from a growth process, we can model this growth as an infinite family of virtual, fractal, and locally irreversible trajectories. This allows to write a growth equation in a form which can be integrated into a Schrödinger-like equation.

The structuring implied by such a generalized Schrödinger equation provides a new basis to study, with a purely energetic approach, the issues of formation, duplication, bifurcation and hierarchical organization of structures (Nottale 2007).

An inspiring example is the solution describing growth from a center, which bears similarities with the problem of particle scattering in quantum mechanics. Searching for some of the simplest solutions (with a central potential and a spherical symmetry), a solution leads to a flower shape, the common Platycodon flower (see Fig. 2). In honor to Erwin Schrödinger, Nottale, Chaline and Grou named their (2009) book “Flowers for Schrödinger” (Des fleurs pour Schrödinger).

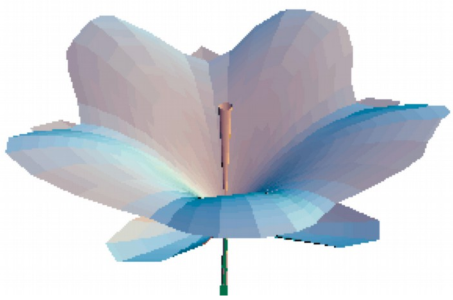

Fig. 2. Schrödinger's flower. Morphogenesis of a flower-like structure, solution of a growth process equation that takes the form of a Schrödinger equation under fractal conditions (from Nottale 2007).

7.3 Biology

In a short paper (Cash et al. 2002), authors inspired by scale relativity proposed a log-periodic law for the development of the human embryo, which fits pretty well with the steps of the human embryo development.

With scale-relativistic models, Nottale and Auffray (Auffray and Nottale 2008; Nottale and Auffray 2008) did tackle the issue of multiple-scale integration in systems biology.

Other studies suggest that many living systems processes, because embedded in a fractal medium, are expected to display wave-like and quantized structuration (Nottale 2013b).

7.4 Geography

The mathematical tools of scale relativity have also been applied to geographical problems (e.g. Forriez, Martin, and Nottale 2010; Nottale, Martin, and Forriez 2012).

7.5 Singularity and evolutionary trees

In their review of approaches to technological singularities, Magee and Devezas (2011, 1370) included the work of Nottale, Chaline and Grou (2000) inspired by scale relativity:

> "Utilizing the fractal mathematics due to Mandlebrot (1983) these authors develop a model based upon a fractal tree of the time sequences of major evolutionary leaps at various scales (log-periodic law of acceleration – deceleration). The application of the model to the evolution of western civilization shows evidence of an acceleration in the succession (pattern) of economic crisis/non-crisis, which point to a next crisis in the period 2015–2020, with a critical point T_c = 2080. The meaning of T_c in this approach is the limit of the evolutionary capacity of the analyzed group and is biologically analogous with the end of a species and emergence of a new species."

The interpretation of this emergence of a new species remains open to debate, whether it will take the form of the emergence of transhumans, cyborgs, superintelligent AI, or a global brain.

8 - Reception and critique

8.1 Scale relativity and other approaches

It may help to understand scale relativity by comparing it to various other approaches to unifying quantum and classical theories.

8.1.1 String theory

Main article: String theory

Although string theory and scale relativity start from different assumptions to tackle the issue of reconciling quantum mechanics and relativity theory, the two approaches need not to be opposed. Indeed, Castro (1997, 275) suggested to combine string theory with the principle of scale relativity:

> "It was emphasized by Nottale in his book that a full motion plus scale relativity including all spacetime components, angles and rotations remains to be constructed. In particular the general theory of scale relativity. Our aim is to show that string theory provides an important step in that direction and vice versa: the scale relativity principle must be operating in string theory."

8.1.2 Quantum gravity

Main article: quantum gravity

Scale relativity is based on a geometrical approach, and thereby recovers the quantum laws, instead of assuming them. This distinguishes it from other quantum gravity approaches. Nottale (2011, 458) comments:

> "The main difference is that these quantum gravity studies assume the quantum laws to be set as fundamental laws. In such a framework, the fractal geometry of space-time at the Planck

> scale is a consequence of the quantum nature of physical laws, so that the fractality and the quantum nature co-exist as two different things.
> In the scale relativity theory, there are not two things (in analogy with Einstein's general relativity theory in which gravitation is a manifestation of the curvature of space-time): the quantum laws are considered as manifestations of the fractality and nondifferentiability of space-time, so that they do not have to be added to the geometric description."

8.1.3 Loop quantum gravity

Main article: Loop quantum gravity

They have in common to start from relativity theory and principles, and to fulfill the condition of background independence.

8.1.4 El Naschie's E-Infinity theory

El Naschie has developed a similar, yet different fractal space-time theory, because he gives up differentiability *and* continuity. El Naschie thus uses a "Cantorian" space-time, and uses mostly number theory (see Nottale 2011, 7). This is to be contrasted with scale relativity, which keeps the hypothesis of continuity, and thus works preferentially with mathematical analysis and fractals.

8.1.5 Causal dynamical triangulation

Main article: Causal dynamical triangulation

Through computer simulations of causal dynamical triangulation theory, a fractal to nonfractal transition was found from quantum scales to larger scales (Loll 2008). This result seems to be compatible with quantum-classical transition deduced in an other way, from the theoretical framework of scale relativity.

8.1.6 Noncommutative geometry

Main article: Noncommutative geometry

For both scale relativity and non-commutative geometries, particles are geometric properties of space-time. The intersection of both theories seems fruitful and still to be explored. In particular, Nottale (2011, 277) further generalized this non-communicativity, saying that it "is now at the level of the fractal space-time itself, which therefore fundamentally comes under Connes's noncommutative geometry (Connes 1994; Lapidus 2008). Moreover, this noncommutativity might be considered as a key for a future better understanding of the parity and CP violations, which will not be developed here."

8.1.7 Doubly special relativity

Main article: Doubly special relativity

Both theories have identified the Planck length as a fundamental minimum scale. However, as Nottale (2011, 459) comments:

> "the main difference between the "Doubly-Special-Relativity" approach and the scale relativity one is that we have identified the question of defining an invariant length-scale as

coming under a relativity of scales. Therefore the new group to be constructed is a multiplicative group that becomes additive only when working with the logarithms of scale ratios, which are definitely the physically relevant scale variables, as we have shown by applying the Gell-Mann–Levy method to the construction of the dilation operator (see Sec. 4.2.1)."

8.2 Cognitive aspects

Special and general relativity theory are notoriously hard to understand for non-specialists. This is partly because our psychological and sociological use of the concepts of space and time are not the same as the one in physics. Yet, the relativity of scales is still harder to apprehend than other relativity theories. Indeed humans can change their positions and velocities but have virtually no experience of shrinking or dilating themselves.

Such transformations appear in fiction however, such as in Alice's Adventures in Wonderland or in the movie Honey, I Shrunk the Kids.

8.3 Sociological analysis

Sociologists Bontems and Gingras (2007) did a detailed bibliometrical analysis of scale relativity and showed the difficulty for such a theory with a different theoretical starting point to compete with well-established paradigms such as string theory.

Back in 2007, they considered the theory to be neither mainstream, that is, there are not many people working on it compared to other paradigms; but also neither controversial, as there is very little informed and academic discussion around the theory. The two sociologists thus qualified the theory as "marginal", in the sense that the theory is developed inside academics, but is not controversial.

They also show that Nottale has a double career. First, a classical one, working on gravitational lensing, and a second one, about scale relativity. Nottale first secured his scientific reputation with many publications about gravitational lensing (e.g. Karoji and Nottale 1976; Nottale and Vigier 1977), then obtained a stable academic position, giving him more freedom to explore the foundations of spacetime and quantum mechanics.

A possible obstacle to the growth in popularity of scale relativity is that fractal geometries necessary to deal with special and general scale relativity are less well known and developed mathematically than the simple and well-known self-similar fractals. This technical difficulty may make the advanced concepts of the theory harder to learn. Physicists interested in scale relativity need to invest some time into understanding fractal geometries. The situation is similar to the need to learn non-euclidian geometries in order to work with Einstein's general relativity (Vidal 2010).

Similarly, the generality and transdiciplinary nature of the theory also made Auffray and Noble (2010) comment: "The scale relativity theory and tools extend the scope of current domain-specific theories, which are naturally recovered, not replaced, in the new framework. This may explain why the community of physicists has been slow to recognize its potential and even to challenge it."

Nottale's (1998a) popular book, written in French, has been compared (Merker 1999, 166) with Einstein's popular book "Relativity: the special and general theory". A future translation of this book from French into English might help the popularization of the theory.

8.4 Reactions

The reactions from scientists to scale relativity are generally positive. For example, Baryshev and Teerikorpi (2002, 255) write:

> Though Nottale's theory is still developing and not yet a generally accepted part of physics, there are already many exciting views and predictions surfacing from the new formalism. It is concerned in particular with the frontier domains of modern physics, i.e. small length- and time-scales (microworld, elementary particles), large length-scales (cosmology), and long time-scales.

Regarding the predictions of planetary spacings, Potter and Jargodski (2005, 113) commented:

> In the 1990s, applying chaos theory to gravitationally bound systems, L. Nottale found that statistical fits indicate that the planet orbital distances, including that of Pluto, and the major satellites of the Jovian planets, follow a numerical scheme with their orbital radii proportional to the squares of integers n^2 extremely well!

Auffray and Noble (2010, 303) gave an overview:

> Scale relativity has implications for every aspect of physics, from elementary particle physics to astrophysics and cosmology. It provides numerous examples of theoretical predictions of standard model parameters, a theoretical expectation for the Higgs boson mass which will be potentially assessed in the coming years by the Large Hadron Collider, and a prediction of the cosmological constant which remains within the range of increasingly refined observational data. Strikingly, many predictions in astrophysics have already been validated through observations such as the distribution of exoplanets or the formation of extragalactic structures.

Although many applications have led to validated predictions (see above), Peter (2013) criticized a provisionally estimated value of the Higgs boson made by Nottale (2011):

> "a prediction for the Higgs boson that should have been observed at mH ≃113.7GeV...it would appear, according to the book itself, that the theory it describes would be already ruled out by LHC data!"

However, this prediction was initially made (Nottale 2001) at a time when the Higgs boson mass was totally unknown. Additionally, the prediction does not rely on scale relativity itself, but on a new suggested form of the electroweak theory. The final LHC result is mH=125.6 ± 0.3 GeV (Beringer, et. al., and (Particle Data Group) 2013, 33), and lies therefore at about 10% of this early estimate.

Particle physicist and skeptic Victor Stenger (2011, 100) also noticed that the theory "predicts a nonzero value of the cosmological constant in the right ballpark." He also acknowledged that the theory "makes a number of other remarkable predictions".

9 - See also

Fractal cosmology
General relativity
Special relativity
Galilean invariance
Multifractal system

10 - Acknowledgments

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11 - References

Abbott, Laurence F., and Mark B. Wise. 1981. “Dimension of a Quantum-Mechanical Path.” *American Journal of Physics* 49 (1): 37–39. http://www.neurotheory.columbia.edu/~larry/AbbottAmJPhys81.pdf.

Allen, A. D. 1983. “Fractals and Quantum Mechanics.” *Speculations in Science and Technology* 6 (2): 165–70.

Anz, Meador P. 2000. “A Decade of Growth.” *The Orbital Debris Quarterly News*, NASA JSC, 5 (4): 1–2. http://orbitaldebris.jsc.nasa.gov/newsletter/pdfs/ODQNv5i4.pdf.

Auffray, C., and D. Noble. 2010. “Scale Relativity: An Extended Paradigm for Physics and Biology?” *Foundations of Science* 16 (4): 303–5. doi:10.1007/s10699-010-9203-x. http://www.arxiv.org/abs/0912.5508.

Auffray, C., and L. Nottale. 2008. “Scale Relativity Theory and Integrative Systems Biology: 1 Founding Principles and Scale Laws.” *Progress in Biophysics and Molecular Biology* 97 (1): 79–114. http://luth2.obspm.fr/~luthier/nottale/arPBMB08AN.pdf.

Baryshev, Yurij, and Pekka Teerikorpi. 2002. *Discovery of Cosmic Fractals*. River Edge, N.J: World Scientific.

Ben Adda, Fayçal, and Jacky Cresson. 2004. “Quantum Derivatives and the Schrödinger Equation.” *Chaos, Solitons & Fractals* 19 (5): 1323–34. doi:10.1016/S0960-0779(03)00339-4.

———. 2005. “Fractional Differential Equations and the Schrödinger Equation.” *Applied Mathematics and Computation* 161 (1): 323–45. doi:10.1016/j.amc.2003.12.031.

Beringer, J., et. al., and (Particle Data Group). 2013. “Status of Higgs Boson Physics.” *PR D86*, no. 010001. http://pdg.lbl.gov/2013/reviews/rpp2013-rev-higgs-boson.pdf.

Bontems, Vincent, and Yves Gingras. 2007. “De La Science Normale a La Science Marginale. Analyse D’une Bifurcation de Trajectoire Scientifique: Le Cas de La Theorie de La Relativité d’Echelle.” *Social Science Information* 46 (4): 607–53. doi:10.1177/0539018407082595. http://ssi.sagepub.com/cgi/content/abstract/46/4/607.

Campesino-Romeo, E., J. C. D'Olivo, and M. Socolovsky. 1982. "Hausdorff Dimension for the Quantum Harmonic Oscillator." *Physics Letters A* 89 (7): 321–24. doi:10.1016/0375-9601(82)90182-7.

Cash, R., J. Chaline, L. Nottale, and P. Grou. 2002. "Développement Humain et Loi Log-Périodique." *Comptes Rendus Biologies* 325 (5): 585–90. doi:10.1016/S1631-0691(02)01468-3. http://luth2.obspm.fr/~luthier/nottale/arcash.pdf.

Castro, Carlos. 1997. "String Theory, Scale Relativity and the Generalized Uncertainty Principle." *Foundations of Physics Letters* 10 (3): 273–93. doi:10.1007/BF02764209. http://link.springer.com/article/10.1007%2FBF02764209.

Célérier, Marie-Noëlle, and Laurent Nottale. 2003. "A Scale-Relativistic Derivation of the Dirac Equation." *Electromagnetic Phenomena* 3: 70–80. http://arxiv.org/abs/hep-th/0210027.

———. 2004. "Quantum–classical Transition in Scale Relativity." *Journal of Physics A: Mathematical and General* 37 (3): 931. doi:10.1088/0305-4470/37/3/026. http://arxiv.org/abs/quant-ph/0609161.

———. 2010. "Electromagnetic Klein–Gordon and Dirac Equations in Scale Relativity." *International Journal of Modern Physics A* 25 (22): 4239–53. doi:10.1142/S0217751X10050615. http://arxiv.org/abs/1009.2934.

Connes, Alain. 1994. *Noncommutative Geometry*. San Diego: Academic Press.

Cresson, Jacky. 2003. "Scale Calculus and the Schrödinger Equation." *Journal of Mathematical Physics* 44 (11): 4907–38. doi:10.1063/1.1618923. http://scitation.aip.org/content/aip/journal/jmp/44/11/10.1063/1.1618923.

Da Rocha, D. 2004. "Structuration gravitationnelle en Relativité d'échelle." Phdthesis, Observatoire de Paris. https://tel.archives-ouvertes.fr/tel-00010204/document.

Da Rocha, D., and L Nottale. 2003. "Gravitational Structure Formation in Scale Relativity." *Chaos, Solitons & Fractals* 16 (4): 565–95. doi:10.1016/S0960-0779(02)00223-0. http://arxiv.org/abs/astro-ph/0310036.

Dubois, Daniel M. 2000. "Computational Derivation of Quantum Relativist Electromagnetic Systems with Forward-Backward Space-Time Shifts." In *AIP Conference Proceedings*, 517:417–29. AIP Publishing. doi:10.1063/1.1291279. http://scitation.aip.org/content/aip/proceeding/aipcp/10.1063/1.1291279.

Dubrulle, B. 2000. "Finite Size Scale Invariance." *The European Physical Journal B - Condensed Matter and Complex Systems* 14 (4): 757–71. doi:10.1007/s100510051087. http://link.springer.com/article/10.1007/s100510051087.

Feynman, Richard P., and Albert R. Hibbs. 1965. *Quantum Mechanics and Path Integrals*. International Series in Pure and Applied Physics. New York: McGraw-Hill.

Forriez, M., P. Martin, and L. Nottale. 2010. "Lois d'échelle et transitions fractal-non fractal en géographie." *L'Espace géographique* Vol. 39 (2): 97–112. http://www.cairn.info/resume.php?ID_ARTICLE=EG_392_0097.

Galileo, Galilei. 1991. *Dialogues Concerning Two New Sciences*. Great Minds Series. Buffalo, N.Y: Prometheus Books.

Gell-Mann, M., and M. Lévy. 1960. "The Axial Vector Current in Beta Decay." *Il Nuovo Cimento* 16 (4): 705–26. doi:10.1007/BF02859738.

Groom, D. E., M. Aguillar-Benitez, C. Amsler, and et. al. 2000. "Review of Particle Physics. Particle Data Group." http://inis.iaea.org/Search/search.aspx?orig_q=RN:31041263.

Herrmann, Felix Johan. 1997. "A Scaling Medium Representation: A Discussion on Well-Logs, Fractals and Waves." PhD thesis, TU Delft, Delft University of Technology. http://repository.tudelft.nl/assets/uuid:5423a23f-814e-4260-ba9b-f60e16cd2f14/as_herrmann_1997.pdf.

Jumarie, G. 2001. "Schrödinger Equation for Quantum Fractal Space–Time of Order N Via the Complex-Valued Fractional Brownian Motion." *International Journal of Modern Physics A* 16 (31): 5061–84. doi:10.1142/S0217751X01005468. http://www.worldscientific.com/doi/abs/10.1142/S0217751X01005468.

———. 2006. "Modified Riemann-Liouville Derivative and Fractional Taylor Series of Nondifferentiable Functions Further Results." *Computers & Mathematics with Applications* 51 (9–10): 1367–76. doi:10.1016/j.camwa.2006.02.001. http://www.sciencedirect.com/science/article/pii/S0898122106000861.

———. 2007. "The Minkowski's Space–time Is Consistent with Differential Geometry of Fractional Order." *Physics Letters A* 363 (1–2): 5–11. doi:10.1016/j.physleta.2006.10.085. http://www.sciencedirect.com/science/article/pii/S0375960106017038.

Karoji, H., and L. Nottale. 1976. "Possible Implications of the Rubin-Ford Effect." *Nature* 259 (5538): 31–33. doi:10.1038/259031a0. http://dx.doi.org/10.1038/259031a0.

Konacki, Maciej, and Alex Wolszczan. 2003. "Masses and Orbital Inclinations of Planets in the PSR B1257+12 System." *The Astrophysical Journal Letters* 591 (2): L147. doi:10.1086/377093. http://iopscience.iop.org/1538-4357/591/2/L147.

Kröger, H. 1997. "Proposal for an Experiment to Measure the Hausdorff Dimension of Quantum-Mechanical Trajectories." *Physical Review A* 55 (2): 951–66. doi:10.1103/PhysRevA.55.951.

Lapidus, Michel L. 2008. *In Search of the Riemann Zeros: Strings, Fractal Membranes and Noncommutative Spacetimes*. Providence, R.I: American Mathematical Society.

Loll, R. 2008. "The Emergence of Spacetime or Quantum Gravity on Your Desktop." *Classical and Quantum Gravity* 25 (11): 114006. doi:10.1088/0264-9381/25/11/114006. http://arxiv.org/abs/0711.0273.

Magee, Christopher L., and Tessaleno C. Devezas. 2011. "How Many Singularities Are Near and How Will They Disrupt Human History?" *Technological Forecasting and Social Change* 78 (8): 1365–78. doi:10.1016/j.techfore.2011.07.013. http://web.mit.edu/cmagee/www/documents/29-singularitysdarticle.pdf.

Mandelbrot, Benoit B. 1983. *The Fractal Geometry of Nature*. Macmillan.

Merker, Joël. 1999. "Deux infinis cousus main." *Revue de synthèse* 120 (1): 165–74. doi:10.1007/BF03182083. http://link.springer.com/article/10.1007/BF03182083.

Müller, Xavier. 2005. “La Relativité d’Echelle - Et Si Le Monde N’était Qu’un Océan Fractal ?” *Science et Vie*.

Nottale, L. 1989. “Fractals and the Quantum Theory of Spacetime.” *International Journal of Modern Physics A* 04 (19): 5047–5117. doi:10.1142/S0217751X89002156. http://www.worldscientific.com/doi/abs/10.1142/S0217751X89002156.

———. 1992. “The Theory of Scale Relativity.” *International Journal of Modern Physics A* 7 (20): 4899–4936. doi:10.1142/S0217751X92002222. http://luth.obspm.fr/~luthier/nottale/arIJMP2.pdf.

———. 1993a. *Fractal Space-Time and Microphysics: Towards a Theory of Scale Relativity*. World Scientific. http://luth2.obspm.fr/~luthier/nottale/LIWOS7-1cor.pdf.

———. 1993b. “Mach’s Principle, Dirac’s Large Numbers and the Cosmological Constant Problem.” Working Paper. http://luth2.obspm.fr/~luthier/nottale/arlambda.pdf.

———. 1994. “Scale Relativity: First Steps Toward a Field Theory.” In *Relativity in General*, 1:121. http://luth2.obspm.fr/~luthier/nottale/arSalas.pdf.

———. 1996a. “Scale Relativity and Fractal Space-Time: Applications to Quantum Physics, Cosmology and Chaotic Systems.” *Chaos, Solitons & Fractals* 7 (6): 877–938. doi:10.1016/0960-0779(96)00002-1. http://luth2.obspm.fr/~luthier/nottale/arRevFST.pdf.

———. 1996b. “Scale-Relativity and Quantization of Extra-Solar Planetary Systems.” *Astronomy and Astrophysics* 315: L9–12. http://luth2.obspm.fr/~luthier/nottale/arA&A315.pdf.

———. 1997. “Scale Relativity.” In *Scale Invariance and Beyond*, edited by B. Dubrulle, F. Graner, and D. Sornette. Berlin, Heidelberg: Springer Berlin Heidelberg. http://luth2.obspm.fr/~luthier/nottale/arhouche.pdf.

———. 1998a. *La Relativité Dans Tous Ses États: Au Delà de l’Espace-Temps*. Hachette.

———. 1998b. “Scale Relativity and Quantization of the Planetary System around the Pulsar PSR B1257 + 12.” *Chaos, Solitons & Fractals* 9 (7): 1043–50. doi:10.1016/S0960-0779(97)00079-9. http://luth2.obspm.fr/~luthier/nottale/arPSR.pdf.

———. 1998c. “Scale Relativity and Quantization of Planet Obliquities.” *Chaos, Solitons & Fractals* 9 (7): 1035–41. doi:10.1016/S0960-0779(97)00078-7. http://luth2.obspm.fr/~luthier/nottale/arobliq.pdf.

———. 2001. “Scale Relativity and Non-Differentiable Fractal Space-Time.” In *Frontiers of Fundamental Physics 4*, edited by B. G. Sidharth and M. V. Altaisky, 65–79. Springer US. http://luth.obspm.fr/%7Eluthier/nottale/arBirla00.pdf.

———. 2003a. “La relativité d’échelle à l’épreuve des faits.” *Pour la Science*. http://www.pourlascience.fr/ewb_pages/a/article-la-relativite-d-echelle-a-l-epreuve-des-faits-24365.php.

———. 2003b. “Scale-Relativistic Cosmology.” *Chaos, Solitons and Fractals* 16 (4): 539–64. http://luth2.obspm.fr/~luthier/nottale/arScRCosmo.pdf.

———. 2004a. *Relativité D’échelle et Structuration de L’univers*. https://www.youtube.com/watch?v=JwaKLTitwtc.

———. 2004b. “The Theory of Scale Relativity: Non−Differentiable Geometry and Fractal Space−Time.” In *AIP Conference Proceedings*, 718:68–95. AIP Publishing. doi:10.1063/1.1787313. http://luth2.obspm.fr/~luthier/nottale/arcasys03.pdf.

———. 2007. “Scale Relativity: A Fractal Matrix for Organization in Nature.” *Electronic Journal of Theoretical Physics* 4 (16): 187–274. http://www.luth.obspm.fr/%7Eluthier/nottale/arEJTP.pdf.

———. 2010. “Scale Relativity and Fractal Space-Time: Theory and Applications.” *Foundations of Science* 15 (2): 101–52. doi:10.1007/s10699-010-9170-2. http://www.arxiv.org/abs/0912.5508.

———. 2011. *Scale Relativity and Fractal Space-Time: A New Approach to Unifying Relativity and Quantum Mechanics*. World Scientific Publishing Company.

———. 2012. “Nature et Valeur de La Constante Cosmologique.” In *Premières Rencontres d’Avignon (2007-2009) Autour de La Relativité d’Echelle*, edited by L. Nottale and P. Martin, 31–39. Actes d’Avignon. Avignon. http://luth.obspm.fr/%7Eluthier/nottale/arAvLambda.pdf.

———. 2013a. Laurent Nottale : La Tête dans les Étoiles Interview by Sophie Sendra. http://www.bscnews.fr/201310283220/Exclusivites/laurent-nottale-la-tete-dans-les-etoiles.html.

———. 2013b. “Macroscopic Quantum-Type Potentials in Theoretical Systems Biology.” *Cells* 3 (1): 1–35. doi:10.3390/cells3010001. http://www.ncbi.nlm.nih.gov/pmc/articles/PMC3980741/.

Nottale, L., and C. Auffray. 2008. “Scale Relativity Theory and Integrative Systems Biology: 2 Macroscopic Quantum-Type Mechanics.” *Progress in Biophysics and Molecular Biology* 97 (1): 115–57. http://luth2.obspm.fr/~luthier/nottale/arPBMB08NA.pdf.

Nottale, L., and M. N. Célérier. 2007. “Derivation of the Postulates of Quantum Mechanics from the First Principles of Scale Relativity.” *Journal of Physics A-Mathematical and Theoretical* 40 (48): 14471–98. doi:10.1088/1751-8113/40/48/012. http://arxiv.org/abs/0711.2418.

Nottale, L., M.N. Célérier, and T. Lehner. 2006. “Non-Abelian Gauge Field Theory in Scale Relativity.” *Journal of Mathematical Physics* 47 (3): 032303. http://arxiv.org/abs/hep-th/0605280.

Nottale, L., J. Chaline, and P. Grou. 2000. *Les Arbres de L’évolution: Univers, Vie, Sociétés*. Hachette littératures.

———. 2002. “On the Fractal Structure of Evolutionary Trees.” In *Fractals in Biology and Medicine*, edited by Gabriele A. Losa, Theo F. Nonnenmacher, Danilo Merlini, and Ewald R. Weibel, 247–58. Birkhäuser. http://luth2.obspm.fr/%7Eluthier/nottale/arbiomed.pdf.

———. 2009. *Des Fleurs Pour Schrödinger: La Relativité d’Echelle Et Ses Applications*. Ellipses.

Nottale, L., P. Martin, and M. Forriez. 2012. “Analyse En Relativité D’échelle Du Bassin Versant Du Gardon (Gard, France).” In *Premières Rencontres d’Avignon (2007-2009) Autour de La Relativité d’Echelle*, edited by L. Nottale and P. Martin, 267–95. Actes d’Avignon. Avignon. http://luth.obspm.fr/%7Eluthier/nottale/arAvGardon.pdf.

Nottale, L., and J. Schneider. 1984. “Fractals and Nonstandard Analysis.” *Journal of Mathematical Physics* 25 (5): 1296–1300. http://luth2.obspm.fr/~luthier/nottale/arFNSAtex.pdf.

Nottale, L., G. Schumacher, and J. Gay. 1997. “Scale Relativity and Quantization of the Solar System.” *Astronomy and Astrophysics* 322: 1018–25. http://luth2.obspm.fr/~luthier/nottale/arA&A322.pdf.

Nottale, L., G. Schumacher, and E. T. Lefevre. 2000. “Scale-Relativity and Quantization of Exoplanet Orbital Semi-Major Axes.” *Astronomy and Astrophysics* 361: 379–87. http://luth2.obspm.fr/~luthier/nottale/arA&A361.pdf.

Nottale, L., and J. P. Vigier. 1977. “Continuous Increase of Hubble Modulus behind Clusters of Galaxies.” *Nature* 268 (5621): 608–10. doi:10.1038/268608a0. http://dx.doi.org/10.1038/268608a0.

Ord, G. N. 1983. “Fractal Space-Time: A Geometric Analogue of Relativistic Quantum Mechanics.” *Journal of Physics A: Mathematical and General* 16 (9): 1869. doi:10.1088/0305-4470/16/9/012. http://iopscience.iop.org/0305-4470/16/9/012.

Peter, Patrick. 2013. “Laurent Nottale: Scale Relativity and Fractal Space-Time.” *General Relativity and Gravitation* 45 (7): 1459–61. doi:10.1007/s10714-013-1535-8.

Planck Collaboration, P. A. R. Ade, N. Aghanim, C. Armitage-Caplan, M. Arnaud, M. Ashdown, F. Atrio-Barandela, et al. 2013. “Planck 2013 Results. XVI. Cosmological Parameters.” *arXiv:1303.5076 [astro-Ph]*, March. http://arxiv.org/abs/1303.5076.

Potter, Frank, and Christopher Jargodzki. 2005. *Mad About Modern Physics: Braintwisters, Paradoxes, and Curiosities*. Hoboken, N.J: J. Wiley.

Sidharth, B. G. 2001. *Chaotic Universe: From Planck to the Hubble Scale*. Contemporary Fundamental Physics. Huntington, N.Y: Nova Science Publishers.

Stenger, V. J. 2011. *The Fallacy of Fine-Tuning: Why the Universe Is Not Designed for Us*. Prometheus Books.

Turner, Philip. 2013. “A Review of Scale Relativity and Fractal Space-Time.” http://www.researchgate.net/publication/259373681_A_Review_of_Scale_Relativity_and_fractal_space-time.

Turner, Philip, and L. Nottale. 2015. “The Origins of Macroscopic Quantum Coherence in High Temperature Super Conductivity.” *Physica C: Superconductivity and Its Applications*. doi:10.1016/j.physc.2015.04.006. http://arxiv.org/abs/1410.3659.

Vidal, C. 2010. “Introduction to the Special Issue on the Evolution and Development of the Universe.” *Foundations of Science* 15 (2): 95–99. doi:10.1007/s10699-010-9176-9. http://www.arxiv.org/abs/0912.5508.

Wolszczan, A., and D. A. Frail. 1992. “A Planetary System Around the Millisecond Pulsar PSR1257 + 12.” *Nature* 355 (6356): 145–47. doi:10.1038/355145a0.

Yao, W.-M., and et. al. 2006. “Review of Particle Physics.” *Journal of Physics G: Nuclear and Particle Physics* 33 (1): 1. doi:10.1088/0954-3899/33/1/001. http://iopscience.iop.org/0954-3899/33/1/001.