ground water

SEEPAGE, a New MODFLOW DRAIN Package

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Abstract

The prediction of the location of ground water discharge areas is a key aspect for the protection and (re)development of ground water-dependent wetlands. Ground water discharge areas can be simulated with MODFLOW using the DRAIN package by setting the drain level equal to the topography, while the conductance is mostly set to an arbitrary high value. However, conceptual and practical problems arise in the calculation of the ground water discharge by the DRAIN package as calculated water tables above the land surface, difficult parameterization of the conductance, and large water balance errors. To overcome these problems, a new SEEPAGE package for MODFLOW is proposed. The basic idea of this package is an adaptable constant head cell. It has a variable head, unless the ground water rises above the seepage level, in which case it has a constant head cell. The estimation of the ground water discharge location along a homogeneous, isotropic, linear sloping profile is used to verify the model and to compare it to the DRAIN solution. In an application to three basins in Belgium, it is shown that the SEEPAGE package can be used in combination with the DRAIN package in situations where an upper boundary for a free water table and additional resistance for drainage is required. It is clearly demonstrated that the identification and delineation of regional ground water discharge areas is more accurate using the SEEPAGE package.

Introduction

Our current understanding of wetlands is insufficient to assess the effects of past and future wetland loss. While knowledge of wetland hydrology is crucial, ground water flows are often neglected or uncertain (Hunt et al. 1996). Ground water inflows to wetlands can be estimated by traditional Darcy's law calculations, stable isotope mass balances, temperature profile modeling, and numerical water balance modeling (Hunt et al. 1996). Stoertz and Bradbury (1989) use the budget calculation of MODFLOW on an interpolated grid to estimate the recharge and discharge areas by interpolating densely measured ground water heads and setting all cells to constant head cells.

Batelaan et al. (1993) describe a ground water model for identification of seepage toward wetlands. The quasi-three-dimensional ground water model's main characteristic is that the ground water table in the unconfined aquifer cannot rise above a maximum allowed level. This upper boundary condition is implemented in the successive over-relaxation solver by checking if every newly calculated head rises above this level. In that case, the head will be reset to that level. The maximum allowed level could be the land surface, or less in the case of a dense drainage network. On the basis of the calculated ground water head, a separation can be made in infiltration, intermediate, and discharge areas. Infiltration areas are where the ground water table is below the maximum allowed level. Intermediate or mixed areas are located where the ground water table is equal to the maximum allowed level, but where there is still a vertical downward flux. Hence, in intermediate areas, part of the net precipitation, i.e., precipitation minus evapotranspiration, infiltrates and the remaining part is drained. In ground water discharge areas, the ground water table is equal to the maximum allowed level and there is an upward seepage, which together with net precipitation is removed by drainage.

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The following question and reply from the GMS Mailing List (Froukh 1998 and Daehler-Wilking 1998, respectively) clearly shows the inconvenience people feel with the MODFLOW concept toward the upper level of an unconfined aquifer.

> **Question:** In the case of a confined aquifer, the top and bottom elevations are top of aquifer and bottom of aquifer. But if the aquifer is unconfined, what does the top elevation correspond to?
>
> **Reply:** In this case, the top elevation of the layer refers to the level of the soil surface, but *is not used* by MODFLOW! (Professionals: Please correct me if I'm wrong!) MODFLOW simply assumes that the soil in the top unconfined layer extends upward to infinity. The result of this is that if your model leads to heads that lie above the soil surface (that is, if your model predicts ponding), then the degree of ponding is overstated by a factor of 1/porosity. You may wonder why MODFLOW makes such a fundamental mistake. I think the reason is that MODFLOW does exactly what it says it does—it models the saturated zone and nothing else.

The purpose of this paper is to present an adapted version of the DRAIN package for MODFLOW, which allows the introduction of an upper boundary (land surface) to the hydrogeological system. First, the original DRAIN package is described, followed by an explanation of the concept of the SEEPAGE package. It is then compared with the DRAIN package by estimating the ground water discharge length in a simple test profile. Finally, the usefulness of the SEEPAGE package is demonstrated by the simulation of regional ground water discharge in three catchments in Belgium.

Methods

DRAIN Package

The DRAIN package (McDonald and Harbaugh 1988) is designed to simulate the effects of features such as agricultural drains. A cross section through a drain cell (Figure 1a) shows the concept of a drain in the model. McDonald and Harbaugh (1988) explain this concept in the following manner. The drain is assumed to run only partially full, so that the head within the drain is approximately equal to the median drain elevation, $d_{i,j,k}$ [L]. The head, $h_{i,j,k}$ [L], computed by the model for cell (i,j,k), is actually an average value for the cell and is normally assumed to prevail at some distance from the drain itself. The drain head, $d_{i,j,k}$, prevails only locally, within the drain; it does not characterize the cell as a whole.

The drain removes water from the aquifer at a rate proportional to the difference between the head in the aquifer and drain elevation. The drainage continues as long as the head in the aquifer is above the drain elevation, but ceases if the head falls below that level. The functioning of the DRAIN package is described by the equation pair (McDonald and Harbaugh 1988)

$$QD_{i,j,k} = CD_{i,j,k}(h_{i,j,k} - d_{i,j,k}) \text{ for } h_{i,j,k} > d_{i,j,k} \quad (1)$$

$$QD_{i,j,k} = 0 \text{ for } h_{i,j,k} \leq d_{i,j,k} \quad (2)$$

where $QD_{i,j,k}$ [L^3/T] is the ground water discharge volume flux from cell (i,j,k) into the drain; $h_{i,j,k}$ [L] is the calculated head in cell (i,j,k); and $d_{i,j,k}$ [L] is the drain elevation. The coefficient $CD_{i,j,k}$ [L^2/T] is a lumped conductance describing all of the head loss between the drain and the region of cell (i,j,k) in which the head $h_{i,j,k}$ is assumed to prevail (Figure 1a). This lumped drain conductance is added to the conductance calculated by the block-centered flow (BCF) package, which is derived from the horizontal and vertical resistance between the node center of the drain cell and the centers of the adjacent nodes. The summed DRAIN and BCF conductance therefore defines the total resistance to flow of the MODFLOW drain cell. The head losses between the drain and its adjacent cells are caused by convergent flow toward the drain, flow through the backfill material of the drain, and flow through the wall of the drain (McDonald and Harbaugh 1988). Figure 1b shows a graph of QD vs. h as follows from Equation 1.

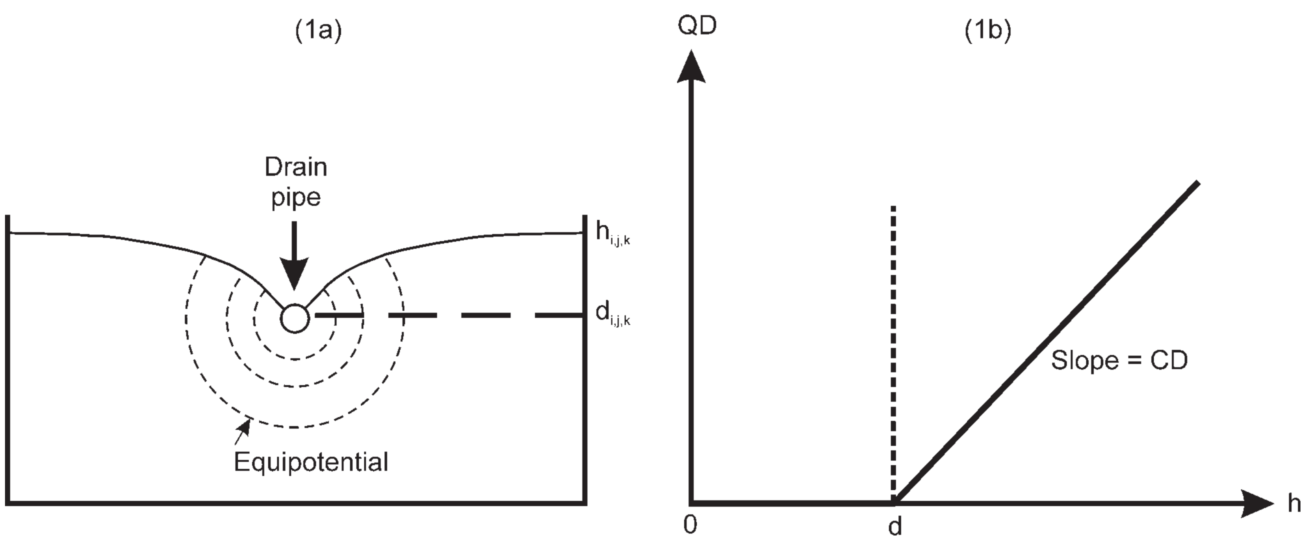

Figure 1. (a) Cross section through cell (i,j,k) illustrating head loss in convergent flow to a drain (after McDonald and Harbaugh [1988]). (b) Plot of flow, QD, into a drain as a function of head, h, in a cell where the elevation of the drain is d and the conductance is CD (after McDonald and Harbaugh [1988]).

Anderson and Woessner (1992) state that springs and seeps can normally be simulated with the DRAIN package by considering the elevation of the spring or seep as it emerges at the land surface as elevation of the drain. In addition, diffuse flows such as seepage to wetlands could be simulated by specifying drain nodes in the area where seepage is likely to occur. The drain nodes will be activated only when the head in the aquifer equals or exceeds the land surface elevation. Conductance terms can be calculated from field-measured discharges and heads or estimated during model calibration. Examples of using MODFLOW's DRAIN package to represent seepage to wetlands can be found in Patrick et al. (1989), Toran and Bradbury (1988), and Yager (1986).

SEEPAGE Package

In order to simulate recharge and discharge areas with MODFLOW, the steady-state flow equation for a phreatic ground water layer is rewritten as

$$\nabla(T\nabla h) + R - S \pm Q = 0 \tag{3}$$

where ∇ is the divergence or gradient operator [L^{-1}]; h is the ground water head [L]; T is the transmissivity [L^2/T], which depends upon h; R is the recharge [L/T], which is positive for inflow; S is the specific ground water discharge or seepage [L/T], which is positive for outflow; and Q is the interactions with the surroundings, e.g., the effects of pumping wells [$L^3/T/L^2$]. However, this equation cannot be solved because both h and S are unknown. Therefore, the area is divided into either recharge or discharge areas; in recharge areas, S is zero and the ground water head can be calculated with Equation 3, whereas in discharge areas, h is set equal to h_S and S can be calculated as

$$S = \nabla(T\nabla h_S) + R \pm Q \tag{4}$$

where h_S [L] is the ground water drainage or seepage level, which can be derived from topography and the presence of discharge features as springs, ditches, marches, rivulets, etc. Hence, the solution procedure consists of determining in an iterative way the position of recharge and discharge areas using the equations given previously such that everywhere $h \leq h_S$ and $S \geq 0$.

Batelaan et al. (1993) developed this technique for simulation of discharge and infiltration areas, and implemented the iterative solution directly in the successive overrelaxation solver of a two-dimensional steady-state finite-difference ground water model. However, in MODFLOW, because of the grouping of hydrological boundary conditions in different packages, a direct implementation in the solver was not feasible. Therefore, a new SEEPAGE package was developed (Batelaan 1999). In this SEEPAGE package, the phreatic water table is limited to a maximum allowed level, e.g., the land surface, and when the water table becomes equal to this level, the resulting ground water discharge (seepage) to the surface occurs and can be calculated with Equation 4.

Three different states for the ground water volume flux are possible, as shown in Figure 2. Here the states are described for the case when recharge is the only stress applied to the seepage cell. The procedure, similar to the one followed by Streng and van Ellen (1997), proceeds by checking for every defined seepage cell whether the calculated head, h, is equal or above the seepage level, h_S. If this is the case, h is put equal to h_S and the ground water volume seepage flux, QS [L^3/T], out of the cell is calculated with the numerical equivalent of Equation 4:

$$QS = \sum_{n=1}^{nn} CD_n\,(h_n - h_S) + RA \pm QA \tag{5}$$

where QS is the product of the specific ground water seepage, S [L/T], and the surface area of the top of the cell through which the seepage takes place A [L^2]; the term, $\sum_{n=1}^{nn} CD_n\,(h_n - h_S)$, represents the ground water volume

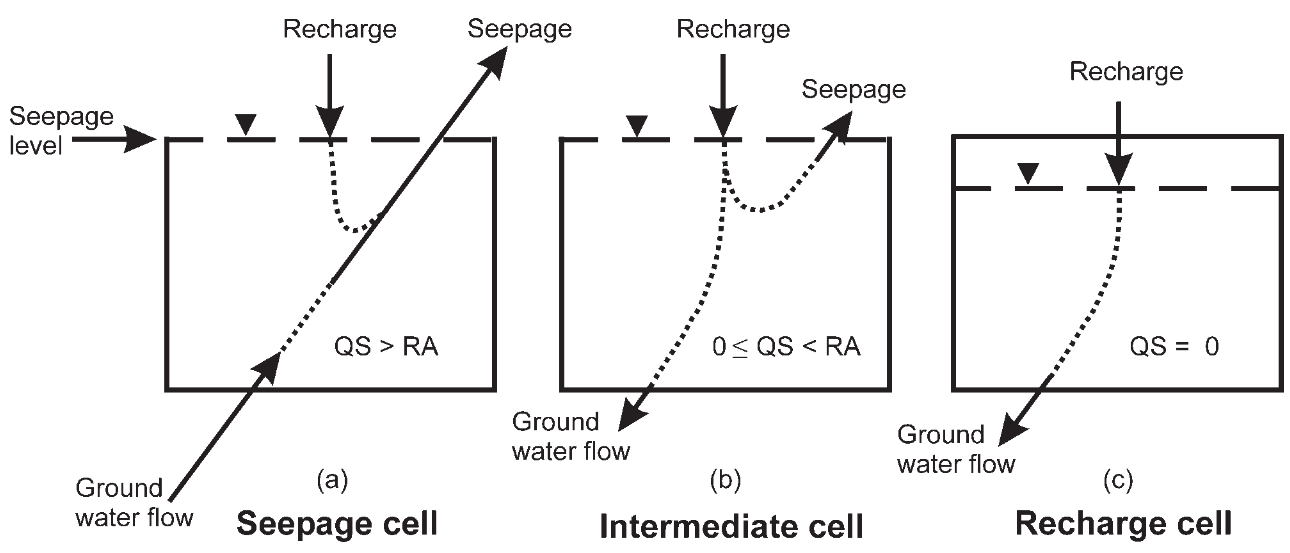

Figure 2. Concept of ground water discharge or (a) seepage, (b) intermediate, and (c) recharge cell. QS and RA are respectively the ground water discharge or seepage flux and recharge volume flux.

flux in (positive) or out (negative) of the cell (*i,j,k*); *nn* is the number of neighbor cells of cell (*i,j,k*); CD_n [L^2/T] is the conductance between cell (*i,j,k*) and its neighbor *n;* h_n [*L*] is the hydraulic head in neighboring cell *n*; *RA* [L^3/T] is the volumetric recharge flux (positive) in cell (*i,j,k*), equal to the product of *R* and *A*; and *QA* [L^3/T], the product of *Q* and *A*, the other volume fluxes in (positive) or out (negative) of the cell, as for instance resulting from pumping or recharging wells. The conductance CD_n is calculated in the BCF package as is the case for the DRAIN package. However, a difference with the DRAIN package is that in SEEPAGE, besides the BCF resistance or conductance, no additional resistance to flow is defined.

If the seepage volumetric flux, *QS,* is larger than the recharge volume flux, *RA*, then the cell is a true seepage cell (Figure 2a). This means that water from sources other than recharge is entering the cell, e.g., ground water inflow from neighboring cells or inflow from other sources, as recharge wells. Consequently, the ground water level*, h*, becomes high, reaching the seepage level, h_S, and excess water is drained from the cell by means of seepage flow. In the SEEPAGE package, the seeping cell is turned into a constant head cell, and the ground water level is fixed at the seepage level. In this case, the defined recharge is to the cell, not to the larger ground water domain, since it will be immediately removed by seepage.

If the ground water seepage volume flux is positive, but smaller than the recharge volume flux, then the cell is intermediate (Figure 2b). This means there is a net ground water outflow from the cell to the neighboring cells or to another sink, for instance, a pumping well. However, this outflow is smaller than the inflowing recharge volume flux, such that the ground water level also rises up to the seepage level and excess recharge water is removed by seepage. In this case, the cell cannot be considered as a true seepage cell, because the seepage flow is smaller than the recharge and part of the recharge still replenishes the ground water reservoir. Also for an intermediate cell, the SEEPAGE package turns the cell into a constant head cell.

A third possibility occurs when, for a presumed seepage cell, the calculated seepage volume flux becomes negative, i.e., there would be an inflowing seepage, which is physically not possible. Hence, the cell is therefore turned into a variable head cell (active node) such that it functions as a recharge cell and, consequently, the ground water seepage will become zero.

In version 2.0 of the SEEPAGE package (Batelaan 1999), other sources and sinks in addition to the recharge are taken into account to compare with the ground water seepage volume flux. Volume fluxes of the well, evapotranspiration, drain, river, flow and head boundary, general head boundary, reservoir, interbed storage, stream, transient leakage, and block-centered flow packages are supported. This means that a summation is performed of the different available stresses in a seepage cell in order to determine the status of the cell (recharge, intermediate, or discharge). This checking of the status of a seepage cell is repeated every iteration and all constant head seepage cells receive a special coding such that they are verified, and hence can become variable again. After convergence, the SEEPAGE performs a budget calculation for every final seepage cell. Ground water discharge or seepage areas are identified from the budget as cells with a negative closure term, i.e., a seepage volume flux (outflow) larger than the sum of stresses in the seepage cell.

Since the SEEPAGE package is based on the DRAIN package (version 5), it is completely compatible with its parent. This means that DRAIN data files of MODFLOW-88 or MODFLOW-96 can be used by the SEEPAGE package since the listed conductance will be ignored. Additionally, SEEPAGE accepts matrices as an input by defining a mask (similar to the IBOUND array) and a seepage level matrix. If the mask value is bigger than zero, the seepage level will be activated.

Test Case

A test case is considered to show the ability of the SEEPAGE package to simulate seepage to wetlands and in order to compare its results and performance with the DRAIN package. Figure 3 shows a cross-sectional flow domain of length *L* = 200 m with a linear sloping land surface from 22 to 20 m (the slope is denoted by *–a*). The phreatic aquifer consists of one layer with a homogeneous isotropic conductivity of 1 m/day and impervious bottom and sides. The aquifer receives a constant recharge of 1.5 mm/day over the whole profile, but seepage is possible all over the land surface. Presumably, in the higher reach of the aquifer, the water table will be lower than the land surface and all recharge will infiltrate. However, in the lower reach of the aquifer, the water table can coincide with the land surface, seepage will occur, and no recharge will reach the ground water. In the intermediate, there can be a small zone where the water table coincides with the land surface and only part of the recharge will infiltrate and reach the ground water. In view of the fact that the horizontal dimension is much larger than the vertical dimension, the ground water flow can be considered as predominantly horizontal. Hence, under the Dupuit-Forchheimer assumption, the flow through any vertical section at position *x* is given by

$$q = -Kh\frac{dh}{dx} \tag{6}$$

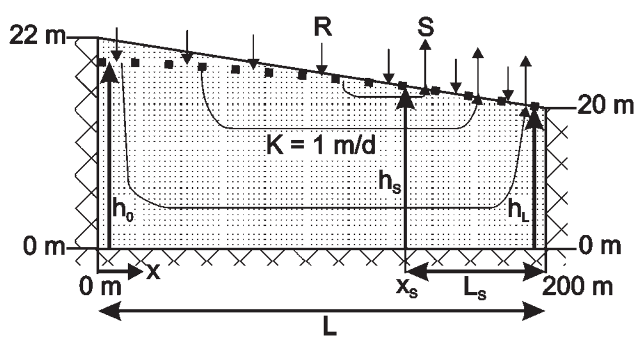

Figure 3. Cross-sectional flow to a ground water discharge area. L and L_S are respectively the length of the profile and the seepage zone; h_0, h_S, and h_L are the ground water heads at respectively x = 0 (left boundary), x = x_S (start of seepage zone), and at x = 200 m (right boundary); K is the conductivity of the aquifer material; R is the recharge, constant defined over the whole profile; and S is the seepage.

where q [L^2/T] is the ground water discharge over the entire aquifer thickness per unit width, and h [L] is the water table elevation measured from the bottom of the aquifer. In the case of steady-state conditions, continuity requires that the discharge equals the recharge amount over length x [L] in the infiltration area. Along the length of the ground water discharge area, L_S [L], the slope of the water table is $dh/dx = -a$. Hence, at the boundary between recharge and discharge areas, the following equations apply:

$$h_S = h_L + aL_S \tag{7}$$

$$h_S Ka = R(L - L_S) \tag{8}$$

where h_S [L] is the water table elevation at the boundary between recharge and discharge areas. This can be solved to yield the length of the discharge zone:

$$L_S = \frac{RL - Kah_L}{R + Ka^2} \tag{9}$$

where h_L [L] is the water table elevation at $x = L$. The assumption of horizontal flow does not allow the identifying of any intermediate zone. Substituting the values for the test profile in Equation 9 results in a length for the ground water discharge area of $L_S = 62.5$ m. By integrating Dupuit-Forchheimer's Equation 6 and finding the integration constant from $x_S = 137.5$ m with water table level $h_S = 20.625$ m, the maximum height of the parabolic shaped water table at $x = 0$ can be calculated as $h_0 = 21.3$ m.

To simulate this problem with MODFLOW, a one-layer model is set up, with cells of 1 by 1 m arranged in one row and 201 columns. The lower and side boundaries are no-flow and all cells receive a recharge of 1.5 mm/day. The initial head is taken as 22.0 m. Several cases were considered as shown in Table 1. In the first seven cases, the DRAIN package is used to simulate the seepage with the drain level set equal to the topography and assuming different conductances. Cases 1 and 4 are simulated with the standard MODFLOW-88 code (McDonald and Harbaugh 1988), while cases 2, 5, and 6 are simulated with the MODFLOW-96 (version 3.2) code (Harbaugh and McDonald 1996a). Cases 3 and 7 are tested with MODFLOW-96 compiled in double precision. In all cases, the solution is obtained with the preconditioned conjugate-gradient 2 (PCG2) solver (Hill 1990) with maximum 50 inner iterations and a head change and residual criterion for convergence set to 10^{-5}.

In order to check the functioning of the SEEPAGE package, the test problem is simulated with SEEPAGE for cases 8, 9, and 10, as shown in Table 1. Case 8 uses MODFLOW-96 (version 3.2) in double precision, while case 9 is tested with single precision. In case 10, a model with 22 layers, as in Streng and van Ellen (1997), is used to simulate the test problem. The layers are horizontal with a thickness of 1 m. In all SEEPAGE cases, the head change and residual criterion for convergence is also set to 10^{-5}.

Results and Discussion

Based on the description of the DRAIN package (McDonald and Harbaugh 1988), it is clear that it was not specifically meant for application of seepage problems and identification of wetlands. In wetlands, relatively large surface areas have a ground water table equal to the land surface. Often the spatial resolution of the model is such that the area of interest, i.e., the wetlands, is covered by many cells. Therefore, over their full area a head should prevail, which is equal to the supposed maximum (drainage) level. This supposition is, however, in contradiction with the assumption of the DRAIN condition that the drain head prevails only locally, within the drain, and that it does not characterize the cell as a whole. This means that the DRAIN assumption leads to a simulated head in the DRAIN cell, i.e., wetlands, which is only valid at some distance from the drain itself (McDonald and Harbaugh 1988). Therefore, the conceptually appealing SEEPAGE condition is introduced, which defines a maximum or seepage level over the entire cell. It is thereby assumed that there is no additional resistance for the exfiltration of ground water at the seepage level. However, if appropriate, it is still possible to introduce

Table 1
Test Cases for DRAIN and SEEPAGE Packages

Case #	Package	Precision	Conductance (m^2/day)	L_S (m)	h_0 (m)	WB error (%)	ITER
1	DRAIN-88	single	1000	—	—	no conv	> 1000
2	DRAIN-96	single	1000	63	21.31	–9.18	199
3	DRAIN-96	double	1000	63	21.31	0.00	187
4	DRAIN-88	single	100	63	21.31	1.19	113
5	DRAIN-96	single	10	64	21.31	0.10	59
6	DRAIN-96	single	1	67	21.31	0.02	29
7	DRAIN-96	double	1	67	21.31	0.00	29
8	SEEP-96	double	n/a	62–63	21.31	0.00	279
9	SEEP-96	single	n/a	62–63	21.31	0.00	279
10	SEEP-96	single	n/a	56–72	21.32	0.00	233

L_S is the calculated extent of the seepage area (when two values are given, these correspond respectively to the length of the seepage area and the seepage face plus intermediate area), h_0 is the maximum ground water level in the infiltration area, WB error is the overall error in the water balance, and ITER is the number of iterations needed for convergence (no conv means that the model failed to converge in < 1000 iterations).

a resistance to exfiltration by combining a DRAIN and SEEPAGE condition in one cell. This might be necessary if the model contains little vertical resolution such that no convergence of flow and corresponding resistance is simulated.

A common misunderstanding of the DRAIN package is that it drains when the water table equals the land surface. In reality, it starts to drain when the water table is slightly above the drain level. The maximum allowed ground water level, introduced in the SEEPAGE package, is similar to the rising water table option in AQUIFEM-1 (Townley and Wilson 1980). However, the method of calculation of the position of the free water table is not rigorously solved in the SEEPAGE package, and neither in MODFLOW (Anderson and Woessner 1992). For example, no deformation of the grid with the moving water table is considered and no nodes are fixed along the water table as is implemented in AQUIFEM-N (Townley 1990). However, Potter and Gburek (1986) showed that incorporating a seepage face into a solution based on Dupuit-Forchheimer assumptions, like in SEEPAGE, is theoretically justified.

If the different test DRAIN cases (Table 1) are compared, it is observed that all single precision MODFLOW cases (1, 2, 4, 5, and 6) have a water balance error > 0.01%. Anderson and Woessner (1992) state that ideally the water balance error should be < 0.1%, while usually an error of ~1% is acceptable. This would mean that cases 5 and 6 would have an acceptable water balance. However, with present-day computers and the more advanced solvers, compared to the ones from the past, models should easily obtain a water balance error < 0.1%. The model for this test case is so small and simple that a water balance error of 0.01%, together with a head convergence of 10^{-5} m points to problems in the model structure or setup leading to bad convergence or low accuracy. Low water balance accuracy mainly results from the very small head differences between the calculated ground water level and the fixed drain level, and the imprecision of the drain flux that results from this. The problem is notified by the warning message at runtime—"IEEE floating-point exception flags raised: inexact; underflow;". The message indicates that the head difference reaches the accuracy of the single precision data representation of the computer. The water balance error is consequently the result of the accumulation of this inaccurate head difference multiplied with the conductance value. Smaller conductance values (cases 4, 5, and 6) cause bigger head differences and therefore result in smaller water balance errors. However, the head still rises up to 4 cm above the drain level for case 6 and the ground water discharge length, L_S, of 67 m, is too large, compared to the theoretical value of 62.5 m calculated with the Dupuit-Forchheimer assumption. Harbaugh and McDonald (1996a, 1996b) recognized the limitations of a single precision budget calculation in MODFLOW-88. They therefore improved the budget calculation of the DRAIN package in MODFLOW-96 to double precision. The improvement is shown by the fact that in case 1 no solution is found due to the lack of precision, while in case 2 a solution is obtained. However, this cannot resolve all inaccuracies in the water balance calculations as is observed in cases 2, 5, and 6. Therefore, it is necessary to convert the complete code to double precision as suggested by Harbaugh and McDonald (1996a). Cases 3 and 7 show that the water balance errors are indeed resolved in MODFLOW-96 with the complete code converted to double precision.

In cases 2, 3, and 4, the ground water discharge length, L_S, is 63 m as identified from the budget of the drain cells. Cases 5, 6, and 7 result in a larger seepage area, 2.5 to 4.0 m more than the analytical solution, which is due to the lower conductances used in the DRAIN package. From the simulated five DRAIN cases with single precision, the lesson learned is that simulation of ground water discharge with the DRAIN package and high conductance (1000 or 100 m^2/day) leads to serious problems as slow, or no, convergence and significant water balance errors. Simulations with low conductance (10 or 1 m^2/day) lead to acceptable water balance errors, but in these cases, MODFLOW calculates a larger ground water discharge extent based on the Dupuit-Forchheimer assumption. This is because low conductances have a significant effect on the ground water flow and hence produce a solution that is different from the analytical solution. It is therefore concluded that a model using DRAIN and high conductance will suffer from water balance errors and while lowering the conductance might resolve water balance errors, it, however, provides larger discharge areas than expected. Cases 3 and 7 show that water balance errors are resolved when MODFLOW is completely converted to double precision. However, the conductance in case 7 appears again too low to describe the discharge condition accurately and results in a too large discharge length.

Test cases 8 and 9 show that both the double and the single precision version of MODFLOW-96 with the SEEPAGE package have no water balance budget problems. The last node at which ground water discharge occurs is at L_S = 62 m. Nevertheless, the water budget of the node at L_S = 63 m shows an outflowing flux smaller than the recharge. Hence, the cell at L_S = 63 m is an intermediate cell with a ground water table equal to the land surface, but still recharge occurs. It can be concluded that with the SEEPAGE package, an accurate ground water discharge length is obtained in single precision, while still maintaining an accurate water balance. The penalty is a higher required number of iterations. All DRAIN and SEEPAGE test cases accurately calculate the water table at the water divide, h_0.

In order to obtain an idea of the effect of the Dupuit-Forchheimer assumption in the previously determined discharge lengths, test case 10 was set up. It consists of 22 horizontal layers, each 1 m thick, with a horizontal and vertical conductivity of 1 m/day. The results of this test case show that with SEEPAGE, the simulated discharge length L_S is 56 m. However, from the SEEPAGE budget calculations, it appears that there is an intermediate area between L_S = 56 m and 72 m. Hence, in this section the water table is equal to the land surface, but still part of the recharge infiltrates. Clearly, allowing vertical flow has the effect of shortening the simulated discharge length as well as creating a much larger intermediate area. It is concluded that vertical discretization (no use of the Dupuit-Forchheimer assumption) has a significant effect on the location of the discharge area. The effect is similar to the DRAIN simulation with low conductance because, in both situations, more resistance to flow is introduced to the system resulting in solving a problem, which is different from the Dupuit-Forchheimer case.

Application

Model Setup

In order to show the practical applicability of the SEEPAGE package, a case study is presented of regional ground water discharge in the Dijle, Demer, and Nete catchments in Belgium (Figure 4). In the past, efforts were made to drain wet valleys for improvement of agricultural production. However, this caused much damage to nature areas and ecologically valuable landscapes. Many wet valleys are fed by discharging ground water typically indicated by wet summer conditions. This seeping ground water forms part of the hydrological cycle. Part of the infiltrating precipitation recharges the aquifers and flows to the topographical lower areas where it seeps out at the land surface as ground water discharge. The discharge areas mostly correspond to the very wet parts of the valleys, where the discharge contributes significantly to the base flow. Both the high ground water levels as well as the chemical composition of the discharging ground water create favorable conditions for ecological development. For nature development and management, it is essential to identify and characterize such ground water discharge areas in function of the hydrogeological characteristics of the region. This characterization enables evaluations of the valleys with regard to maintaining or developing valuable ecosystems and therefore helps in nature planning.

The total model area covers more than 5000 km^2 (Figure 4), mostly located in Flanders, the northern region of Belgium. Large ground water reservoirs are present, consisting of mainly Tertiairy water-bearing formations, slightly sloping toward the north. The thickness of the unconfined part of the water bearing strata was determined from 1285 drillings and mapping the top of the confining formations, which are mostly thick clay layers (Figure 5). The northern part of the study area consists of the thick sands of the Campine aquifer system, underlain by the Boom clay aquitard. A small part in the north of the Dijle Basin consists of the Oligocene sand aquifer system underlain by the confining clay layer of the Formation of Asse. In a larger part of the Dijle Basin, the aquifer system is made of the sand of Brussels and Cretaceous formations, which are confined either by the clay of Kortrijk or by the Paleozoic basement. In the southern part of the Demer Basin, the Paleocene aquifer system is located on top of the Paleozoic basement.

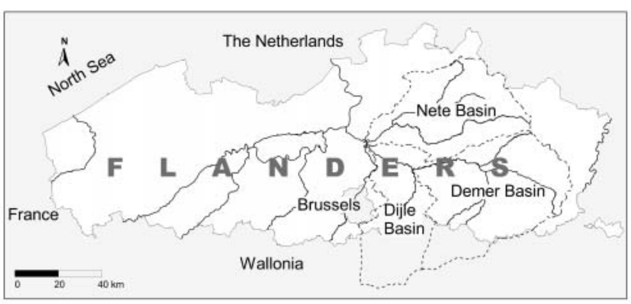

Figure 4. Location of the study area: Dijle, Demer, and Nete basins.

Hydraulic conductivity values for the phreatic ground water systems were taken from De Smedt (1975), Loy and De Smedt (1978), and Bronders (1989). The conductivities were spatially extrapolated within each hydrogeological unit and combined with an isopach map into a transmissivity map of the phreatic aquifer. There are 1529 wells located in the study area with a total yearly average extracted ground water volume of 120.4 10^6 m^3/yr for the period 1993–1995. The specified recharge in the model was calculated with the WetSpass model (Batelaan and De Smedt 2001). The areal averaged WetSpass calculated recharge is 251 mm/yr. A digital elevation model of the National Geographic Institute has been used for the topography of the land surface and the maximum ground water level (Batelaan and De Smedt 1994). All spatial data were discretized with a resolution of 50 by 50 m. The numerical solution was obtained with the PCG2 solver (Hill 1990) with a head criteria of 10^{-3} and residual criteria of 10^{-2}.

To simulate the discharge areas with MODFLOW, the DRAIN and SEEPAGE packages were used in a combined way. The first advantage of this method was that the discharge level could be constrained between a minimum level defined by the DRAIN condition and a maximum level defined by SEEPAGE. The drain level was set equal to 0.5 m below the topography, i.e., the average depth of the ditches in the area, while the seepage level was fixed equal to the topography. The second advantage was that the DRAIN package allowed the introduction of additional resistance to the ground water flow for the combined DRAIN-SEEPAGE cell. The BCF package defines the resistance between the combined DRAIN-SEEPAGE cell and its adjacent cells, while the DRAIN package adds to this the local drainage resistance in the DRAIN-SEEPAGE cell itself. In the case of a one-layer model, as used here, this additional resistance is necessary to take into account the resistance caused by converging flow toward the drainage location. No additional resistance will result in too small ground water discharge areas because in a one-layer model, the ground water flow is only horizontal and vertical flow is ignored.

Results and Discussion

The results of the model are regional ground water levels, ground water discharge locations, and seepage fluxes. In addition, recharge areas connected to specific ground water discharge areas together with the flow times and flowpaths can be determined with the help of the particle-tracking model MODPATH (Pollock 1994).

The most important result of the model is a ground water discharge map (Figure 6), showing the spatial distribution of the discharge and recharge areas in the three basins. It is the first map that displays, with a high resolution, the spatial distribution of regionally determined ground water flow and the resulting discharge for a large part of Flanders. The ground water discharge areas cover 18.1%, and 6.0% are intermediate areas. The ground water discharge is classified as low discharge < 2.0 mm/day, medium 2.0 to 5.0 mm/day, and high > 5.0 mm/day, covering respectively 8.7%, 7.0%, and 2.4% of the area (Table 2). Together with

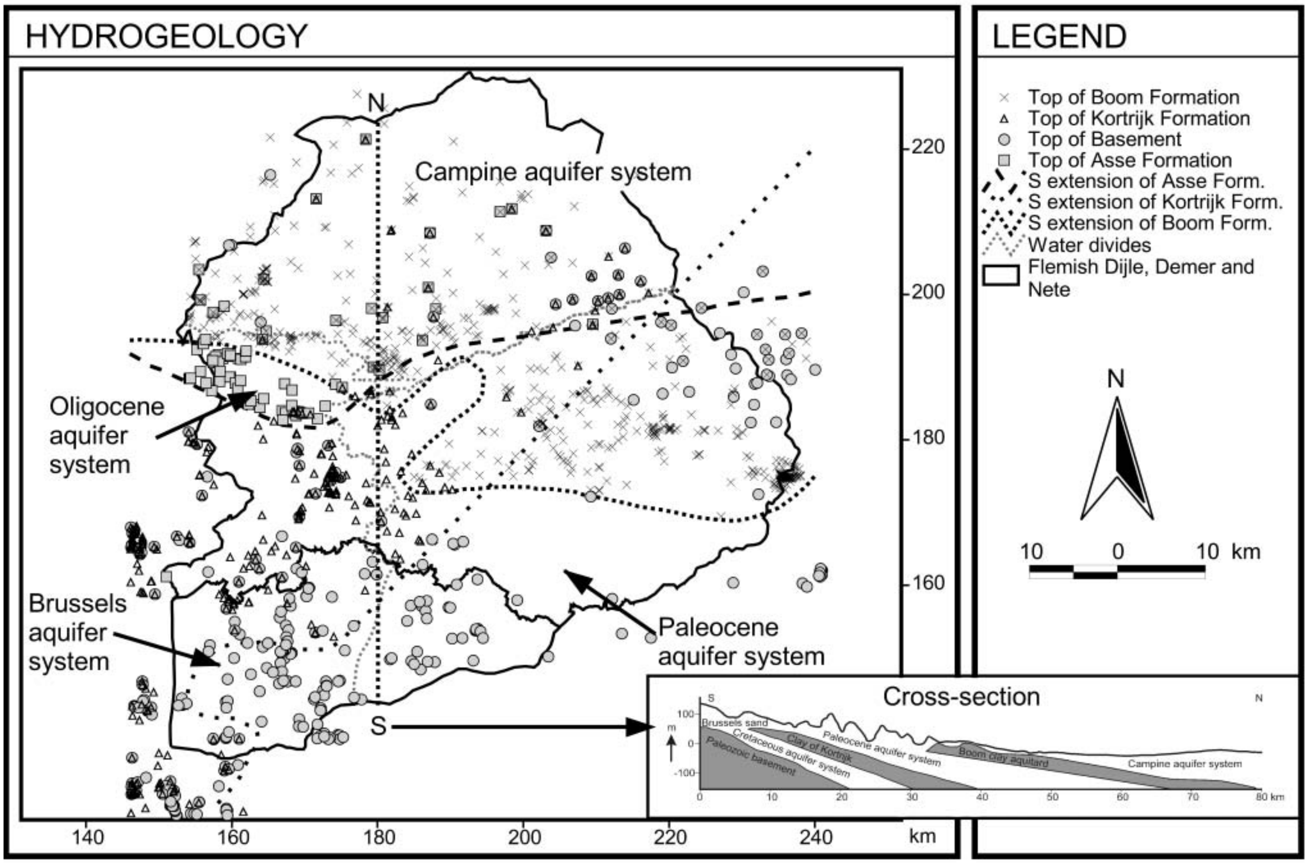

Figure 5. Hydrogeology of study area.

the ground water discharge flux, the surface area of ground water discharge determines the total volume and contribution of ground water to the river runoff in each basin.

The main recharge areas are located on the interfluves, while the discharge areas are constrained in the middle and lower reaches of the river valleys. Generally, the upper reaches contain much less discharge area. In these areas, there is a strong differentiation between extensive recharge areas and localized discharge areas with a high discharge intensity. Here, ground water flows over relatively large distances before it reaches a discharge area. In the downstream part of the valleys, at the confluence area of Dijle and Nete, the situation is much more complex. Here, the ground water discharge and recharge areas are much more tied together, the flow distances much shorter, and the ground water discharge more spatially distributed, which results in lower discharge intensities. Batelaan et al. (2003) showed that for the Grote Nete subcatchment, the simulated discharge patterns and intensities fit very well with the occurrence and types of phreatophytes.

Table 2
Different Types of Recharge and Discharge Classes, Corresponding Range of Recharge or Discharge Flux, Average and Standard Deviation (SD) of the Simulated Flux and the Classified Area in km^2 and as % of the Total Area

Type	Range (mm/day)	Average (mm/day)	SD	Area km^2	%
Recharge		0.7	0.2	3245.4	75.8
Intermediate		0.4	0.2	257.6	6.0
Low discharge	< 2	1.3	0.3	373.5	8.7
Medium discharge	2–5	3.2	0.8	303.9	7.0
High discharge	> 5	10.0	18.0	102.7	2.4

Areas with a high discharge intensity are therefore mainly found in the upper parts of the valleys, as in the southern part of the Demer-Dijle Basin, where the seepage occurs mostly in narrow zones almost coinciding with the river courses themselves. In the northern part of the Demer and Dijle, the discharge areas are much broader as a result of the flatter topography. In addition, the ground water flow systems are more local, resulting in smaller discharge intensities.

Areas with medium ground water discharge are mainly located in the middle parts of the basins. They often cover the complete valley and produce large volumes of ground water discharge. Areas with low discharge intensities occur mainly in the confluence zone of Nete and Dijle, where the ground water discharge results from relatively local and shallow subsurface flow due to the shallow presence of aquitards. Similarly, zones with low ground water discharge intensities occur in a number of smaller tributary valleys in the upper Demer Basin, due to the shallow occurrence of the aquitard of the Formation of Boom.

Model Performance

The targets for calibration of the ground water model are measured phreatic levels in 175 piezometers. For all these piezometers, long, reliable time series are available,

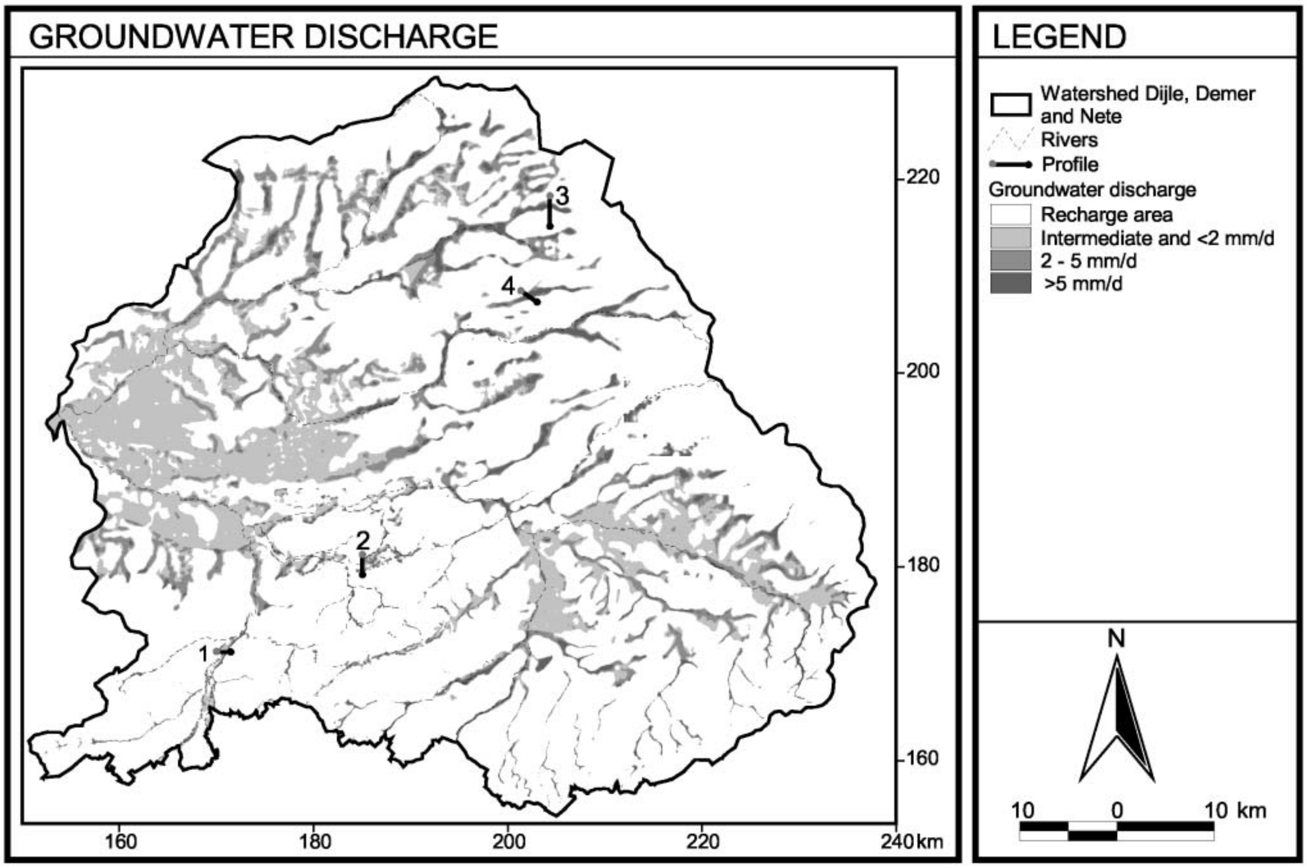

Figure 6. Simulated ground water discharge in Dijle, Demer, and Nete catchments; indicated are recharge, intermediate, and low discharge areas (< 2 mm/day), medium discharge (2 to 5 mm/day), and high discharge (> 5 mm/day).

such that accurate time-averaged phreatic water levels can be determined. Figure 7 shows the measured vs. the simulated phreatic levels, with a correlation coefficient equal to 0.99, indicating a good agreement between model results and field data. The error between simulated and measured levels appears to be spatially homogeneous, i.e., no trend or spatial structure is revealed with higher or lower than average errors. Figure 8 gives the mean error (*ME*, [*L*]), mean absolute error (ME_{abs}, [*L*]), root mean square error (*RMSE*, [*L*]), relative root mean square error ($RMSE_{rel}$, [–]), Nash-Sutcliffe model efficiency (*NS*, [–]), and coefficient of determination (*CD*, [–]), which are respectively defined as

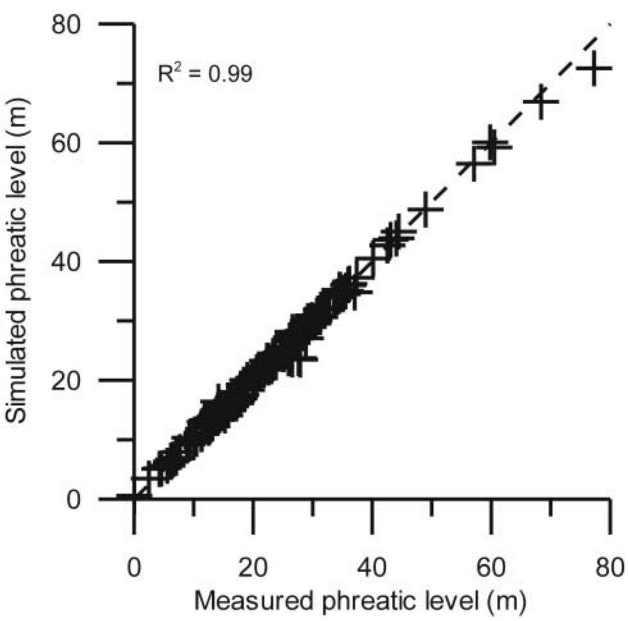

Figure 7. Comparison of measured and simulated phreatic ground water levels for Dijle, Demer, and Nete catchments.

$$ME = \frac{\sum_{1}^{n}(h_m - h_o)}{n} \tag{10}$$

$$ME_{abs} = \frac{\sum_{1}^{n}[abs(h_m - h_o)]}{n} \tag{11}$$

$$RMSE = \sqrt{\frac{\sum_{1}^{n}(h_m - h_o)^2}{n}} \tag{12}$$

$$RMSE_{rel} = \frac{1}{\bar{h}_o}\sqrt{\frac{\sum_{1}^{n}(h_m - h_o)^2}{n}} \tag{13}$$

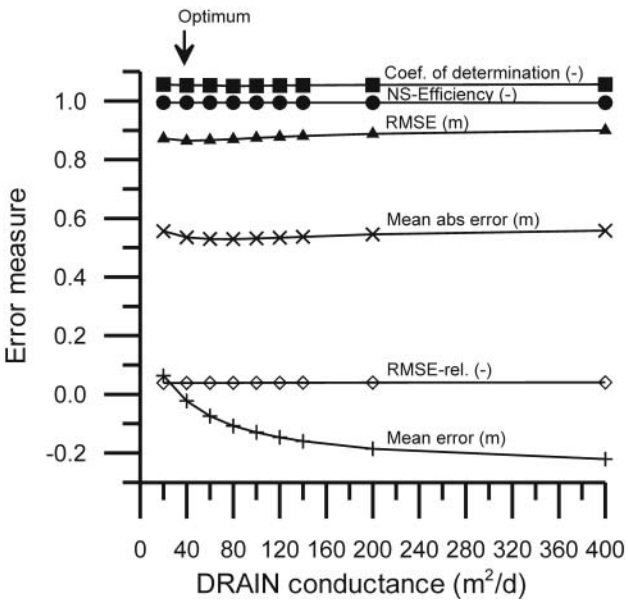

Figure 8. Calibration of the ground water model with respect to the DRAIN conductance as measured by mean error (m), mean absolute error (m), root mean square error (m), relative root mean square error (–), model efficiency (–), and coefficient of determination (–).

$$NS = 1 - \frac{\frac{1}{n}\sum_{1}^{n}(h_m - h_o)^2}{\frac{1}{n}\sum_{1}^{n}(h_o - \bar{h}_o)^2} \tag{14}$$

$$CD = 1 - \frac{\frac{1}{n}\sum_{1}^{n}(h_o - \bar{h}_o)^2}{\frac{1}{n}\sum_{1}^{n}(h_m - \bar{h}_o)^2} \tag{15}$$

where h_o [L] is the observed head, h_m [L] is the modeled head, $\bar{h}_o$[L] is the average of the ground water head observations, and n is the total number of measurements. The *NS* (Nash and Sutcliffe 1970; Gupta et al. 1998) expresses the ability of the model to reproduce the measurements. Generally, a model can be considered unsatisfactory if the efficiency factor is < 0.5, while it can be considered good if the efficiency is > 0.9. The *CD* describes the relation of the scattering of the simulated and observed values around the mean of the observations. Good model results should yield values close to 1.

Since the drain conductance of the DRAIN-SEEPAGE combined model cannot be measured, it has been calibrated in the ground water model. It was varied between 20 and 400 m^2/day, as shown in Figure 8, where indicators of the performance of this calibration are represented. The *ME* was lowest, –0.02 m, for a conductance of 40 m^2/day, indicating that a very small systematic error is present. The ME_{abs} is in this case 0.53 m and nearly equal to its minimum value of 60 m^2/day. The *RMSE* had a minimum of 0.86 m for a conductance of 40 m^2/day. The $RMSE_{rel}$ shows an almost constant value of 0.04, with a slight minimum for a conductance of 40 m^2/day. The $RMSE_{rel}$ value is close to the ideal zero value. The same holds for the *NS*, which shows a high model efficiency of 0.99 for all runs. The *CD* is practically constant with values between 1.05 and 1.06. The *CD* indicates that there is slightly less scattering of the simulated phreatic level around the observed mean than for the measured values.

As observed from this calibration, the most informative parameter is the *RMSE*, since it most clearly shows a minimum value consistent with the other parameters. It is concluded that the optimal calibrated model exists for a drain conductance of 40 m^2/day. The good calibration result is likely due to some extent by the constant head nodes that are enforced by the SEEPAGE package. The effect for the calibration is that this will limit the error in the simulated heads within the range of the uncertainty of the topographic elevation of the discharging parts of the valleys. This will also constrain the head solution in the recharging parts of the catchment. Besides head calibration, Batelaan et al. (2003) show that there is also good correspondence with field data for the fluxes, i.e., the recharge is in balance with the measured river base flow, and the summation of the simulated surface runoff and recharge from WetSpass are within 2% of the total measured river discharge.

The necessity for also using the SEEPAGE package is investigated by comparing the DRAIN-SEEPAGE combined model runs with a run in which only DRAIN was used. In addition, in this case it appears that 40 m^2/day is the optimal conductance. However, the simulated ground water head is now up to a maximum 3 m higher than the head of the combined DRAIN-SEEPAGE calibrated model. The head over the model area is, on average, 0.10 m higher. In comparison to the topography, this means that the head for 39.4 km^2 of the valley areas is up to a maximum 3.0 m above the topography, with the average being 0.26 m. Since the drainage level was defined as 0.5 m below the topography, the ground water level rises in the lowest parts of the valleys at locations of high ground water discharge 0.76 m above its drainage base. A consequence of these higher ground water levels is that the total discharge areas also increase with 2.4% or 24.8 km^2.

The conclusion for this calibration is that the model simulates well the actual situation. Nevertheless, it is necessary to state that this calibration is for a supraregional model with regional piezometric information. This means that on a local scale, the model might perform worse in simulating actual ground water levels. This is especially true since, at the local scale, not all drainage characteristics are accurately known and included in the model. The model therefore rather indicates the potential of discharge more than the actual situation.

Phreatic Level Profiles

Figure 9 shows four profiles, two in the Nete, one in the Demer, and one in the Dijle Basin, indicating the topography, the ground water levels simulated with the combined DRAIN-SEEPAGE, and with only the DRAIN package. The four profiles show typical ground water discharge situations in valleys. In profile 1, in the Dijle Valley, the two simulated ground water levels are reaching the defined

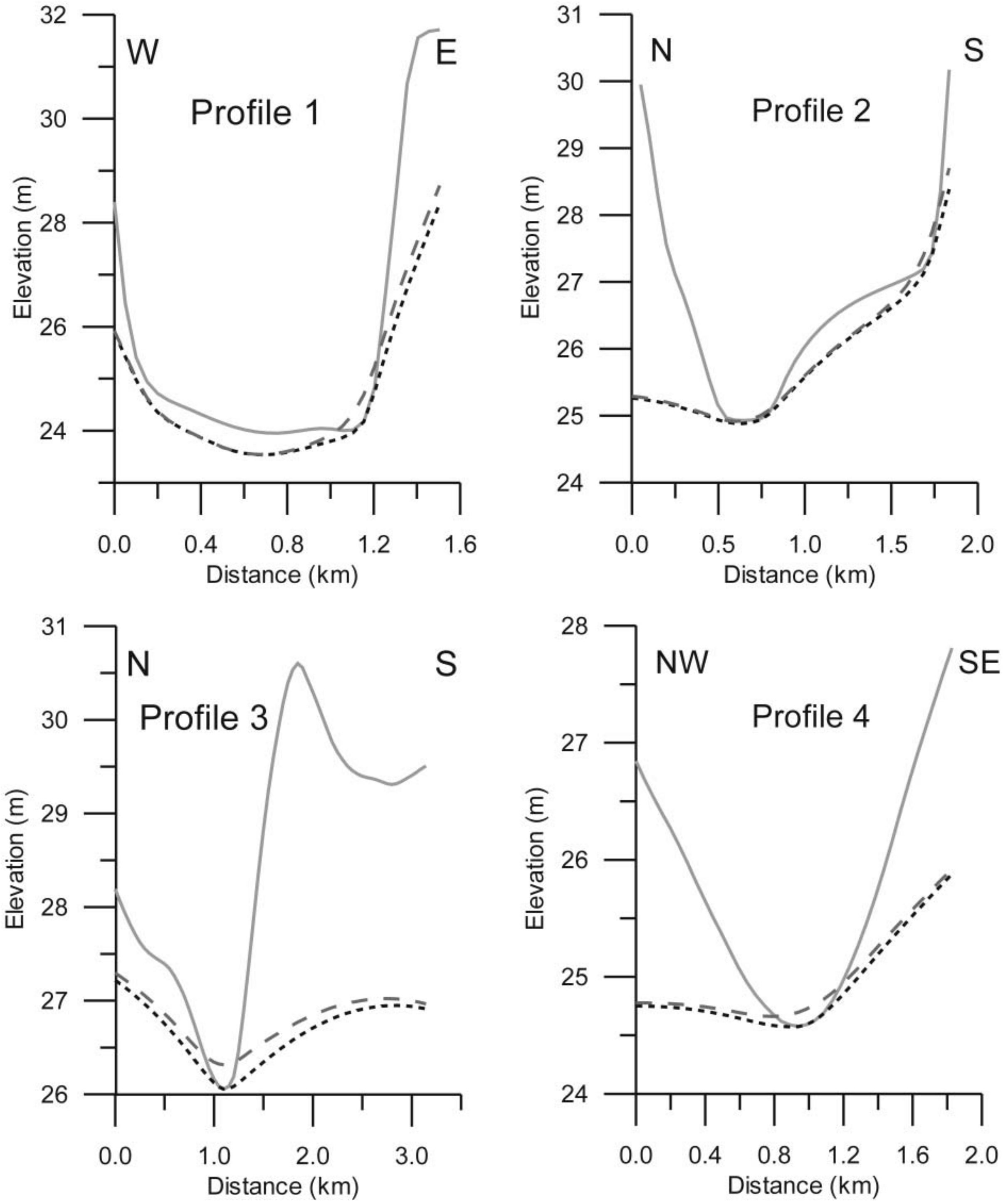

Figure 9. Four profiles (for their location, see Figure 6) showing the topography (solid line), the simulated ground water level determined with the combined DRAIN-SEEPAGE package (short-dashed line), and with only the DRAIN package (long-dashed line).

ground water drainage level of 0.5 m below the topography for most of the relatively broad valley. The ground water discharge is very high on the eastern side of the valley floor. Due to the large flux of ground water discharge at the foot of the hillslope, the ground water levels are above the defined drainage level for both types of simulations. For the simulation with the DRAIN package, the phreatic ground water level reaches an unrealistic level above the topography. On the other hand, with the DRAIN-SEEPAGE simulation, the ground water level is located exactly at the defined seepage level equal to the topography.

In the second north-south profile in the Demer Basin, the phreatic level at the foot of the southern slope is above the topography for the simulation with only the DRAIN condition. Due to the large ground water flux from the south to the valley and a strong break in the slope at the foot of the southern valley side, it appears that without SEEPAGE the ground water level is not able to follow the strong land surface curvature. In the middle part of the valley, the topography shows a strong convex profile such that the ground water level is not reaching the land surface. In the northern part of the valley, the ground water level coincides again with the topography but, in the case of the DRAIN simulation, is even slightly above the topography.

The third, north-south, profile in the Nete Basin consists of a narrow valley with a very strong topographic curvature. The strongly converging ground water flow results in a very localized high ground water discharge flux. In this case, the simulation with only DRAIN results in high ground water levels, which are everywhere above the valley floor and also much too high outside the valley.

The fourth profile depicts a V-shaped valley located in an upper reach of the Nete Basin. The deep incised valley causes a steep ground water gradient with a high discharge flux, especially on the southeast side. As can be observed from the figure, the phreatic ground water level simulated with the combined DRAIN-SEEPAGE nicely follows the land surface.

The model-predicted locations of the ground water discharge areas are most likely sensitive to the grid cell size (Haitjema et al. 2001; Lepoutre, 2002). Although no test was performed on the optimal grid cell size for this area, it can be determined on the basis of Haitjema et al. (2001) that the grid cell size should be smaller than the characteristic leakage length as defined by Haitjema et al. (2001), which, in our case, is ~150 m. Since in this application a grid cell size of 50 m was used, no significant resolution effect on the predicted ground water discharge patterns or area size is to be expected.

In general, the DRAIN simulated ground water levels can be forced closer to the topography by increasing the DRAIN conductance locally in the valleys. However, a higher DRAIN conductance is not realistic with respect to local drainage conditions. Moreover, higher DRAIN conductances will make the numerical solution less stable and can cause large computational errors. Therefore, from the comparison between the DRAIN and DRAIN-SEEPAGE simulations, it is concluded that regional ground water discharge areas are more accurately and realistically simulated by using SEEPAGE.

Conclusions

The prediction of the location of the recharge, intermediate, and discharge areas in the landscape is a problem, which is of high importance for ecological development or restoration projects. Biodiversity, and therefore ecological value, is often high in areas where there is constant soil wetness due to discharging ground water with a lithotrophic water quality. Ground water modeling is an appropriate tool for mapping these areas. In MODFLOW, the most logical choice for simulating ground water discharge areas is the use of the DRAIN package. However, its use results in several practical problems as poor convergence, too highly calculated water tables in and close to the discharge areas, large water balance errors, and difficult parameterization of the conductance. From the original documentation of the MODFLOW code, it is also clear that the DRAIN package was not designed with the aim of determination of recharge and discharge areas.

A new MODFLOW SEEPAGE package is therefore proposed in which the main feature is an adaptable constant head cell. This type of cell is a constant head cell if the water table is above the seepage level; otherwise, it is a variable head cell. From the comparison of the seepage volume flux with the recharge and (or) other applied stresses, the status of the cell (i.e., recharge, intermediate, or discharge) can be determined. The input to the SEEPAGE package is also improved over the DRAIN package because not only list-type, but also matrix-type, input is possible. This allows using digital terrain models as the upper limit of the unconfined aquifer with a minimum of preprocessing. The results of the SEEPAGE budget will show the location of ground water discharge, intermediate, and recharge areas.

A comparison of the SEEPAGE and DRAIN packages on a simple test profile shows that the DRAIN package often results in an overestimation of the ground water discharge length. It is also concluded that it is advisable to use the DRAIN package with a MODFLOW code completely converted to double precision accuracy. On the other hand, the SEEPAGE package is able to accurately predict the ground water discharge length in single precision, while also keeping an accurate water balance. However, a solution based on the SEEPAGE package needs more iterations than a solution based on the DRAIN package. It is shown that by increasing the number of layers in the profile, and therefore the accuracy by which the vertical flux component is simulated, the extent of the ground water discharge increases considerably as well as the size of the intermediate zone.

In an application of a large-scale regional ground water model for three catchments in Belgium, it is demonstrated that the SEEPAGE and DRAIN package can also be combined. The SEEPAGE package functions mainly as an upper boundary of the phreatic aquifer, while the DRAIN package can be used to add resistance to the exfiltration in the one-layer model. The model was calibrated with respect to the conductance. From different error measures, the *RMSE* measure shows most clearly the minimum error. The resulting ground water discharge map for the three simulated catchments shows a large variation in discharge to the valleys, strongly influenced by the geomorphology of the catchments. Evaluation of the discharge with respect to the present vegetation will help in determining the potential for ecological restoration and development. The regional simulated phreatic ground water level with the SEEPAGE-DRAIN package was also compared to a solution obtained by using only the DRAIN package. In four profiles (Figure 9) located in valleys, it was clearly shown that the DRAIN solution generally produces water tables that are too high in, and close to, the discharge areas. Combining the SEEPAGE with the DRAIN package results in an obvious improvement for prediction of the phreatic water levels in the valleys.

It is concluded that the SEEPAGE package is useful as the upper limit for an unconfined aquifer, as an identification tool for estimation of recharge, intermediate, and discharge areas, and for hypothesis testing of ground water discharge to actual or potential wetlands. The SEEPAGE package is fully documented and available from *http://homepages.vub.ac.be/~batelaan/*.

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