The Dimension Problem for Groups and Lie Rings
THOMAS SICKING, Georg-August-Universität Göttingen, Germany

The dimension subgroup problem can be stated as follows: Take a group $G$ and its integral group ring $\mathbb{Z}G$. Let $\epsilon : \mathbb{Z}G \to \mathbb{Z}$ be the augmentation map, i.e. the linear extension of the map $g \mapsto 1$ to $\mathbb{Z}G$ and set $\Delta(G) = \ker(\epsilon)$. Then the group $D_n(G) := (1 + \Delta(G)^n) \cap G$ is a normal subgroup of $G$, and one easily sees that $\gamma_n(G)$ is always contained in $D_n(G)$. However, for $n \geq 4$, there are groups with $\gamma_n(G) \neq D_n(G)$.

For Lie rings, an analogous study has been initiated in [1], where similar results to those known in group rings were found. Furthermore, in [2] it is shown, that for a metabelian Lie ring $L$ we have $2D_n(L) \subseteq \gamma_n(L)$, which is stronger than any result known for metabelian groups so far.

References