Braces were introduced by Rump to study non-degenerate involutive set-theoretic solutions of the Yang-Baxter equation. It has been recently proved that, given a left brace $B$, one can construct explicitly all the non-degenerate involutive set-theoretic solutions of the Yang-Baxter equation such that the associated permutation group is isomorphic, as a left brace, to $B$. It is hence of fundamental importance to describe all simple objects in the class of finite left braces. A left brace $B$ is simple if it is nonzero and $\{0\}$ and $B$ are the only ideals of $B$. In this talk I will explain the matched product decompositions of an arbitrary finite left brace and how to construct new families of finite simple left braces using external matched products of left braces, corresponding to its internal matched product decomposition.

(Joint work with David Bachiller, Eric Jespers, and Jan Okniński.)