On the structure monoid and algebra of left non-degenerate set-theoretic solutions to the Yang–Baxter equation

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Let $r : X^2 \to X^2$ be a set-theoretic solution of the Yang–Baxter equation on a finite set $X$. Denote $M(X, r)$ for the structure monoid $\langle x \in X \mid xy = uv \text{ if } r(x, y) = (u, v) \rangle$. For a finite involutive non-degenerate solution $(X, r)$ of the Yang–Baxter equation it is shown by Gateva-Ivanova and Van den Bergh that the structure monoid $M(X, r)$ is a monoid of I-type, and the structure algebra $K[M(X, r)]$ over a field $K$ shares many properties with commutative polynomial algebras, in particular, it is a Noetherian PI-domain that has finite Gelfand–Kirillov dimension. Motivated by recent work of Lebed and Vendramin for non-degenerate set-theoretic solutions and some results of Jespers and myself for solutions associated to semi-braces, we will discuss the structure monoid for bijective left non-degenerate set-theoretic solutions and its associated algebra over a field $K$. Using a realization of Lebed and Vendramin of $M(X, r)$ as a regular submonoid in the semidirect product $A(X, r) \rtimes \text{Sym}(X)$, where $A(X, r)$ is the structure monoid of the derived solution associated to $(X, r)$, we will show that $K[M(X, r)]$ is a finite module over a central affine subalgebra. In particular, $K[M(X, r)]$ is a Noetherian PI-algebra of finite Gelfand–Kirillov dimension bounded by $|X|$.

Moreover, we will characterize, in ring-theoretical terms of $K[M(X, r)]$, when $(X, r)$ is an involutive solution. This characterization provides, in particular, a positive answer to a recent conjecture of Gateva-Ivanova concerning the cancellativity of $M(X, r)$.

We will relate the prime spectra of the monoids $M(X, r)$ and $A(X, r)$ and show that for bijective square-free left non-degenerate solutions a complete description can be given. Moreover, we will discuss that prime ideals of $M(X, r)$ are determined by the divisibility structure of $M(X, r)$. As the apotheosis of this talk, we will use these results to give a description of the prime spectrum of the structure algebra $K[M(X, r)]$.

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