Classification of groups up to isomorphism is one of the main themes in group theory. A particular challenge is to understand finite \( p \)-groups, that is, groups of \( p \)-power order for a prime \( p \). Since a classification of \( p \)-groups by order seems out of reach for large exponents \( n \), other invariants of groups have been used to attempt a classification; a particularly intriguing invariant is coclass. A finite \( p \)-group of order \( p^n \) and nilpotency class \( c \) has coclass \( r = n - c \).

Recent work in coclass theory is often concerned with the study of the coclass graph \( G(p, r) \) associated with the finite \( p \)-groups of coclass \( r \). The vertices of the coclass graph \( G(p, r) \) are (isomorphism type representatives of) the finite \( p \)-groups of coclass \( r \), and there is an edge \( G \rightarrow H \) if and only if \( G \) is isomorphic to \( H/\gamma(H) \) where \( \gamma(H) \) is the last non-trivial term of the lower central series of \( H \). It is known that one of the feasible approaches for investigating \( G(p, r) \) is to first focus on so-called skeleton groups.

Our definition of skeleton groups for the graph \( G(p, r) \) is based on the technical definition of ‘constructible groups’ given by Leedham-Green. For odd \( p \), let \( S \) be an infinite pro-\( p \) group and informally skeleton groups can be described as twisted finite quotients of \( S \), where the twisting is induced by some suitable homomorphism. Understanding these groups is a crucial step towards investigating the structure of \( G(p, r) \). The first major step towards this is to investigate when two skeleton groups are isomorphic. We present some general results towards this direction including two interesting special cases.

This is a joint work with Dr. Heiko Dietrich, Monash University, Australia.