Let $N$ be a normal subgroup of $G$, $G'$ a subgroup of $G$, and $N'$ a normal subgroup of $G'$. We assume that $N' = G' \cap N$ and $G = G'N$, hence $G := G/N \simeq G'/N'$. Let $b \in Z(\mathcal{O}N)$ and $b' \in Z(\mathcal{O}N')$ be $G$-invariant block idempotents. We denote $A := b\mathcal{O}G$ and $A' := b'\mathcal{O}G'$. Then $A$ and $A'$ are strongly $G$-graded algebras, with 1-components $B$ and $B'$ respectively. Additionally, assume that $C_G(N) \subseteq G'$, and denote $C := \mathcal{O}C_G(N)$, which is regarded as a $G$-graded $G$-acted algebra.

In [2, Definition 2.7.], Britta Späth considers a relation $\geq_c$ between the character triples $(G, N, \theta)$ and $(G', N', \theta')$, where $\theta$ is $G$-invariant irreducible character belonging to the block $b$ and $\theta'$ is a $G'$-invariant irreducible character belonging to the block $b'$.

We introduce $G$-graded $(A, A')$-bimodules over $C$ and we study Morita equivalences between $A$ and $A'$ induced by such bimodules.

We prove that if $\theta$ corresponds to $\theta'$ under a $G$-graded Morita equivalence over $C$, then $(G, N, \theta) \geq_c (G', N', \theta')$.

We also show that an analogue of the so-called “butterfly theorem” [2, Theorem 2.16] holds for $G$-graded Morita equivalences over $C$.

References