New set-theoretical solutions of the pentagon equation
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The pentagon equation is profusely investigated in the modern mathematical physics and it plays an essential role in the development of harmonic analysis on quantum groups. Moreover, it appears in various contexts and with different terminologies, for instance, if $\mathcal{H}$ is a Hilbert space, a unitary operator from $\mathcal{H} \otimes \mathcal{H}$ into itself is said to be multiplicative if it satisfies the pentagon equation.

Our attention is posed on the study of set-theoretical solutions on a set $M$, that are maps $s : M \times M \rightarrow M \times M$ satisfying the relation

$$s_{23} s_{13} s_{12} = s_{12} s_{23},$$

where $s_{12} = s \times id_M$, $s_{23} = id_M \times s$ and $s_{13} = (id_M \times \tau)s_{12}(id_M \times \tau)$ with $\tau$ the flip map on $M \times M$ given by $\tau(x, y) = (y, x)$. First examples of solutions have been provided in the pioneering works by Zakrzewski (1992), Baaj and Skandalis (2003), and Kashaev and Reshetikhin (2007).

In this poster, we show the complete description of set-theoretical solutions of the form $s(x, y) = (x \cdot y, \theta_x(y))$ where $(M, \cdot)$ is a group and $\theta_x$ is a map from $M$ into itself, for every $x \in M$, contained in $\mathcal{H}$. Moreover, we introduce a new technique to construct set-theoretical solutions of the famous quantum Yang-Baxter equation by using solutions of the pentagon equation.

References