Blocks of defect 1 and units in integral group rings

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The Prime Graph Question for integral group rings asks whether it is true that whenever the normalized unit group $V(\mathbb{Z}G)$ of an integral group ring $\mathbb{Z}G$ of a finite group $G$ contains an element of order $pq$, where $p$ and $q$ are different primes, also $G$ must contain an element of order $pq$. This question is known to have a reduction to almost simple groups.

Employing Brauer’s theory of blocks of defect 1 we show that when the Sylow $p$-subgroup of $G$ has order $p$ then $V(\mathbb{Z}G)$ contains an element of order $pq$, for any prime $q$, if and only if $G$ contains an element of order $pq$. This answers the Prime Graph Question in particular for 23 sporadic simple groups.

This is joint work with M. Caicedo.

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