Character degrees in $\pi$-separable groups
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The relation between ordinary characters and Brauer characters is one of the main topics in the Theory of Characters of Finite Groups. This is probably why, in 1974, Martin Isaacs studied how to lift the Brauer characters of a $p$-solvable group, i.e., how to find a family of irreducible ordinary characters which coincide with the irreducible Brauer characters if restricted to the $p$-regular elements.

Later, Isaacs decided to find a more general result. In 1982, in the article *Character of $\pi$-Separable Groups*, using a similar technique he constructed a family of lifts for something like Brauer characters, but generalized for a set of primes instead of just one prime. We will talk about this family of $B_\pi$-characters. After a brief introduction to the theory of $B_\pi$-characters, we will focus on the properties of the degrees of this family of characters $B_\pi(G)$ and on their relation with the group structure.

As expected, in a $\pi$-separable groups the the set of $B_\pi$-character degrees presents some qualities which are already known for the degrees of irreducible Brauer characters. From the set of $B_\pi$-character degrees, in fact, we can tell if the group has a normal $\pi$-complement.

What was not expected is that, if we consider the family $B_\pi(G) \cup B_{\pi'}(G)$, i.e., the family of characters which are either $B_\pi$-characters or $B_{\pi'}$-characters, then the degrees of characters in this family share some properties with the degrees of characters in the whole set $\text{Irr}(G)$. The famous Theorem of Itô-Michler, for example, still holds if we consider the primes which divide not the degrees of some characters in $\text{Irr}(G)$, but just the degrees of some characters in $B_\pi(G) \cup B_{\pi'}(G)$, even if in general this second set is smaller.

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