Quasi-linear left cycle sets and the Retraction Problem
MARCO CASTELLI and GIUSEPPINA PINTO, Università del Salento, Italy

In 2014, Rump introduced the algebraic structure of quasi-linear left cycle sets, a useful framework to work with the set-theoretic solutions of the Yang-Baxter equation. A left cycle set \((A, \cdot)\) is called \textit{quasi-linear} if \(A\) is endowed with an additive abelian group structure such that
\[
a \cdot (b + c) = a \cdot b + (a - b) \cdot c
\]
for all \(a, b, c \in A\). For a quasi-linear left cycle set \(A\), Rump introduced two ideals, the \textit{Socle} and the \textit{Radical} of \(A\), indicated by \(\text{Soc}(A)\) and \(\text{Rad}(A)\) respectively, and another classical tool, the \textit{Fixator} of \(A\), denoted by \(\text{Fix}(A)\). In \cite{1} we show a connection between the quasi-linear left cycle sets with \(\text{Rad}(A) \subseteq \text{Soc}(A)\) and the \textit{unitary metahomomorphisms} on abelian groups, introduced by Gu (1997) in order to find new solutions of the Yang-Baxter equation.

In this poster, we give a characterization of the quasi-linear left cycle sets \(A\) with \(\text{Rad}(A) \subseteq \text{Soc}(A)\) via unitary metahomomorphisms and we give a complete description of those for which \(\text{Rad}(A) \subseteq \text{Fix}(A) = \text{Soc}(A)\) improving the results obtained by Catino and Miccoli (2015). Moreover, we consider a problem posed by Rump (2014) involving the Retract Relation and the related partial results obtained in \cite{1}.

References