Groups, Rings and Associated Structures 2019

Spa, Belgium | June 09-15, 2019

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We gratefully acknowledge funding by
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Practical Information:

Registration: Sunday 17:00 – 19:00 and Monday 08:15 – 08:45.

A full-board reservation at Sol Cress includes the following:
Breakfast: 07:30 – 09:00
Lunch: 12:45, except Wednesday: 13:00.
Dinner: 19:00.

On Thursday the Conference Dinner will take place at 19:00, before there is a conference drink at 18:00.

On Wednesday afternoon there are no lectures. You may optionally take part in a walking tour through the city of Spa (there are two of them, one starting at 14:00 and the other one starting at 16:00). We will also organize a hike through the surroundings of Spa. After dinner there is also a tasting of Belgian beers at around 20:30 in the lecture hall. Please indicate during registration if you want to participate in these activities.

From the railway station "Spa" it is a 10 minutes walk to the cable car (operating on Sunday and Monday until around 21:00) which takes you to the thermal bath right next to the conference venue.

Taxi Jean-Yves in Spa can be reached at +32 495 77 20 20 or +32 87 25 75 51.

Coordinates of the conference site:
Domain Sol Cress
Spaumont 5
4900 Spa
Belgium
Braces were introduced by Rump to study non-degenerate involutive set-theoretic solutions of the Yang-Baxter equation. To study non-involutive solutions one needs skew braces, a non-commutative analog of braces. In this talk, we discuss the structure of skew braces and how properties of skew braces give results on the combinatorial version of the Yang-Baxter equation.

(joint work with S. Koshitani and B. Lancellotti) Based on several theorems concerning the theory of cohomological Mackey functors it is possible to show that every left $B$-lattice $L$, where $B$ is an $OG$-block of a finite group $G$ of cyclic defect group $D = df(B)$, fits into a short exact sequence

$$
0 \rightarrow T_1 \rightarrow T_0 \rightarrow L \rightarrow 0
$$

for some trivial source left $B$-modules $T_1$ and $T_0$. The map $p : \text{Lat}(B) \rightarrow \text{Ts}(B)$, $p([L]) = [T_0] - [T_1]$, can be interpreted as a kind of character from the Grothendieck ring of left $B$-lattices to the Grothendieck ring of trivial source left $B$-lattices. In many significant situations one can compute its values using the splendid derived equivalences introduced by J.Rickard and R.Rouquier.

In the talk we intend to present part of the theory which is necessary for the definition of the character $p$, as well as some immediate applications.

For finitely generated residually finite groups, the growth of the residual girth (the size of finite quotients required to detect $n$-balls of the Cayley graph) is a useful invariant with geometric and computational applications.

We define and study a naturally analogous invariant for finitely generated residually finite dimensional algebras. We analyze several examples, showing that the residual girth growth of group algebras might significantly differ from their group counterparts, and are highly sensitive to the base field structure. We show that for various families of residually finite dimensional algebras the residual girth grows like the usual growth (in the sense of Gelfand-Kirillov). In particular, we prove this for finitely generated prime PI algebras (using a noncommutative Hironaka decomposition argument). This is far from being true in general: we construct finitely generated algebras with GK-dimension 2 having arbitrarily fast residual girth growth.

The talk is based on a work in progress, partially joint with Khalid Bou-Rabee.
How can (modular) representation theorist help ring theory?

Geoffrey Janssens, Vrije Universiteit Brussel, Belgium

Given a set $\mathcal{A}$ of algebras, over a field $F$, a natural problem is to discover which algebras from $\mathcal{A}$ are (not) isomorphic. A classical way to attack such 'distinguishing problems' is my means of invariants. In case of a finite dimensional algebra one can associate to it the so-called codimension sequence $(c_n(A))_n$. This sequence quantify the number of multilinear polynomials (not) satisfied by $A$. In case $F$ has characteristic 0 the sequence $(c_n(A))_n$ grows asymptotically as the function $f(n) = cn^d$ where the constants $c$ and $d$ are half-integers tightly connected to the ring theoretical structure of $A$. In this talk we will focus on explaining the connection between the asymptotics of the codimension sequence and $S_n$-representation theory over $F$. Hereby the goal will be to sketch all problems arising (on both sides of the picture) in case $\text{char}(F) > 0$.

Blocks of defect 1 and units in integral group rings

Leo Margolis, Vrije Universiteit Brussel, Belgium

The Prime Graph Question for integral group rings asks whether it is true that whenever the normalized unit group $V(ZG)$ of an integral group ring $ZG$ of a finite group $G$ contains an element of order $pq$, where $p$ and $q$ are different primes, also $G$ must contain an element of order $pq$. This question is known to have a reduction to almost simple groups.

Employing Brauer’s theory of blocks of defect 1 we show that when the Sylow $p$-subgroup of $G$ has order $p$ then $V(ZG)$ contains an element of order $pq$, for any prime $q$, if and only if $G$ contains an element of order $pq$. This answers the Prime Graph Question in particular for 23 sporadic simple groups.

This is joint work with M. Caicedo.

Property (FA) of the unit group of 2-by-2 matrices over an order in a quaternion algebra

Ann Kiefer, Vrije Universiteit Brussel, Belgium

We study property (FA) and its hereditary version for unit groups of 2-by-2 matrices over orders in totally definite quaternion algebras with rational centres. In particular we consider the three matrix rings over totally definite rational quaternion algebras that can appear as Wedderbrun-Artin components of a group ring $QG$.

$n$ – UJ Rings

Tülay Yıldırım, Gebze Technical University, Turkey

The focus of this article is the rings $R$ for which $u - u^n$ belongs to the Jacobson radical for all units $u$ of $R$, where $n \geq 1$ is a fixed integer. Such rings are called $n$-UJ rings. The structures of these rings are known if $n = 1$ or 2. As well as, we study the $n$-UJ property under some algebraic construction, the trivial extension and the Morita context of $n$-UJ rings are obtained. This recent result with a joint work Tamer Koşan, Truong Cong Quynh and Jan Žemlička [4] among these lines will be presented.

References

[4] M. T. Koşan, Truong Cong Quynh, Tülay Yıldırım and Jan Žemlička, Rings in which the form $u - u^n$ of units belongs to the Jacobson radical, Algebra Colloquium, in press.
on abelian subcategories of triangulated categories
Markus Linckelmann, City, University of London, UK

There are two types of triangulated categories which arise routinely in modular representation theory of finite group algebras - derived module categories, and stable module categories.

derived module categories determine many fundamental numerical invariants of block algebras, such as the number of isomorphism classes of simple modules and irreducible characters of prescribed heights. Broue’s abelian defect conjecture predicts the nature of the derived categories of blocks of finite groups with abelian defect groups.

By contrast, it is not known whether stable module categories determine any of the above mentioned numerical invariants in general. This is one of the major obstacles in modular representation theory. Unlike derived categories, stable module categories need not have any t-structures. Nonetheless, a stable module category has in general a variety of abelian subcategories whose exact structure is compatible with the triangulated structure, and whose numerical invariants are, in some cases, those of the original algebra. We will describe some construction principles for abelian subcategories of stable module categories.

Group rings and the RS property
Sugandha Maheshwary, Indian Institute of Science Education and Research, Mohali, India

Given a group $G$, let $U(Z[G])$ be the group of units of the integral group ring $Z[G]$ and let $Z(U(Z[G]))$ be its center. Trivially, $Z(U(Z[G]))$ contains $\pm Z(G)$, where $Z(G)$ denotes the center of $G$. In case $Z(U(Z[G])) = \pm Z(G)$, i.e., all central units of $Z[G]$ are trivial, the group $G$ is called a cut-group or a group with the cut-property. In 1990, Ritter and Sehgal gave a characterization for finite cut-groups, which was later generalized to arbitrary groups, by Dokuchaev, Polcino Milies and Sehgal. We say that an element $x \in G$ of finite order has the RS-property (or is an RS-element) in $G$ if

$$x^j \sim_G x^{ \pm 1} \text{ for all } j \in U(o(x)),$$

where $o(x)$ denotes the order of $x$, $U(n) := \{ j : 1 \leq j \leq n, \gcd(j, n) = 1 \}$, and $y \sim_G z$ denotes $y$ is conjugate to $z$ in $G$. Let $\Phi(G)$ denote the FC-subgroup of $G$, i.e., the subgroup consisting of those elements of $G$ which have only finitely many conjugates in $G$, and $\Phi^+(G)$ its torsion subgroup. Then the characterization of cut-groups given by Dokuchaev et al can be stated to say that $G$ is a cut-group if, and only if, every element of $\Phi^+(G)$ has the RS-property in $G$. In this talk, I shall present recent results on arbitrary (not necessarily finite) cut-groups, obtained exploring the RS-property.
Low-dimensional representations of finite orthogonal groups
Gunter Malle, Technische Universität Kaiserslautern, Germany

The knowledge of the irreducible characters of finite simple groups of small degree is important for many applications. It turns out that in many cases the second smallest non-trivial degrees are of the order of magnitude of the square of the smallest non-trivial degrees. This was known to hold for about one half of the finite simple groups. We report on an extension of this to most of the remaining cases. This is joint work with Kay Magaard.

Poster Session
(see from page 22 onwards for abstracts)

Coffee Break

Group Rings and Amalgamated Products: The (FA) Property
Doryan Temmerman, Vrije Universiteit Brussel, Belgium

In this talk (GRAP), we will be discussing the geometric property (FA) for groups. This property states that any action of the group on a tree will have a global fixed point. This seems to be an obstruction for the group to have a decomposition as amalgamated product and as HNN extension and is also implied by (for example) Kazhdan’s Property (T). In his famous book “Trees”, J.P. Serre proved that property (FA) can be completely described algebraically.

Using this fact, in recent work with A. Bächle, G. Janssens, E. Jespers and A. Kiefer, we analyzed this geometric property for $U(ZG)$, the units of the integral group ring of a finite group. In this talk we will show how to reduce this problem to certain low-rank special linear groups. An example of such a group, which we were able to analyze completely, is $SL_2(Z[\sqrt{-3}])$. However, our techniques are also applicable to $SL_2$ over some important non-commutative rings, namely orders of totally definite quaternion algebras with rational center.
Donovan’s conjecture predicts that given a $p$-group $D$ there are only finitely many Morita equivalence classes of blocks of group algebras with defect group $D$. While the conjecture is still unknown for a generic $p$-group $D$, the conjecture has been proven in 2014 by Eaton, Kessar, Külshammer and Sambale when $D$ is an elementary abelian 2-group, and in 2018 by Eaton and Livesey when $D$ is any abelian 2-group. The proof, however, does not describe these equivalence classes explicitly. A classification up to Morita equivalence over a complete discrete valuation ring $O$ has been done, among other examples, for $D$ with rank 3 or less, and for $D = (C_2)^4$.

I have classified blocks of finite groups with defect group $D = (C_2)^5$ over an algebraically closed field $k$, and in this talk I will introduce the topic, give the relevant definitions and describe the process of classifying these blocks.

11:45  

A New Family of Decomposition Numbers of the Symmetric Group  
DIEGO MILLAN BERDASCO, Queen Mary, University of London, UK

One of the earliest results in the (still) open problem of describing the $p$-decomposition matrix of the symmetric group was given in 1976 by G.D. James via a map on the set of integer partitions which he named “regularisation”. In this talk we will generalise James’ result by giving a family of $i$-regularisations on partitions, which will give us new decomposition numbers.

12:00  

Some versions of Schur-Weyl duality and respective strength over arbitrary commutative rings  
TIAGO CRUZ, Stuttgart University, Germany

The classical case of Schur-Weyl duality states that the actions of the group algebras of $GL_n$ and $S_d$ on the $d$th-tensor power of a free module, $V^\otimes d$, of finite rank centralize each other. In 2015, Krause showed that when Schur-Weyl duality holds the canonical functor between the category of modules over the Schur algebra $S_R(n,d)$ and the category of homogeneous polynomial representations of degree $d$ of $GL_n$ is an equivalence of categories. Recently, I proved that Schur-Weyl duality holds for commutative rings with certain properties. In the same paper, I showed that another version of Schur-Weyl duality holds for any commutative ring. Namely, the centralizer of $S_R(n,d)$ in $\text{End}_R(V^\otimes d)$ is exactly $RS_d$ for $n \geq d$ for any commutative ring $R$. This fact is generally known for fields, especially for infinite fields. This version of Schur-Weyl duality is related to the Schur functor $F = \text{Hom}_{S_R(n,d)}(V^\otimes d, -)$ and we will make some new comments about the exactness of the right adjoint of $F$.

12:15  

Skew lattices and set-theoretic solutions of the Yang-Baxter equation  
CHARLOTTE VERWIMP, Vrije Universiteit Brussel, Belgium

Finding all set-theoretic solutions of the Yang-Baxter equation is a popular and fundamental open problem of the past years. Recently introduced algebraic structures, called braces and cycle sets, are related to special classes of solutions. In an attempt to describe more general classes of solutions, skew braces and semi-braces were defined. Still, an algebraic structure that describes all set-theoretic solutions of the Yang-Baxter equation is not known. In this 10 MINUTS talk, we discuss set-theoretic solutions obtained from skew lattices, an algebraic structure that has not yet been related to the Yang-Baxter equation. Such solutions are degenerate in general, and thus different from solutions we obtained from braces and other structures. This is yet another step closer to describing all set-theoretic solutions of the Yang-Baxter equation.

This is joint work with Karin Cvetko-Vah.
The twisted group ring isomorphism problem over fields
Ofir Schnabel, Ort Braude College, Israel

Similarly to how the classical group ring isomorphism problem (GRIP) asks, for a commutative ring \( R \), which information about a finite group \( G \) is encoded in the group ring \( RG \), the twisted group ring isomorphism problem (TGRIP) asks which information about \( G \) is encoded in all the twisted group rings of \( G \) over \( R \).

In this talk we will introduce the (TGRIP) and give some results over fields. We will show that the results can be very different than the results for the (GRIP) and discuss some of the methods we used studying this problem.

Joint work with Leo Margolis.

On torsion units of integral group rings – “post-ZC”
Wolfgang Kimmerle, Universität Stuttgart, Germany

The object of the talk are properties of torsion units and torsion subgroups of integral group rings which hold for larger classes of finite groups than the Zassenhaus conjectures.

In particular the question is considered whether a torsion unit of \( ZG \) is conjugate to a trivial unit \( \pm g \in G \) in a group ring \( QH \), where \( H \) is a finite group containing \( G \) as subgroup.

This holds for Sylow tower groups and thus in particular for finite supersoluble groups. The second part of the talk deals with possible Sylow like theorems in the unit group of integral group rings, cf. [2],[3] and [4].

References

G-theory of group rings for finite groups
Iuliia Semikina, University of Bonn, Germany

The \( G \)-theory of a ring \( R \) is defined as Quillen’s \( K \)-theory of the category of finitely generated \( R \)-modules. We will review the Hambleton-Taylor-Williams decomposition conjecture for \( G \)-theory of the integral group rings. The conjecture expresses \( G_n(ZG) \) as a direct sum of the groups \( G_n \) of maximal orders in the simple components of \( QG \) with certain integers inverted, where \( G \) is a finite group.

Using the results of Keating we explicitly know the ranks of both sides of the conjecture for \( G_1 \) and show that the solvable group \( SL(2, F_3) \) is a counterexample to the conjectured decomposition. Nevertheless, we will also talk about related positive results that still hold for all finite groups and explain the connection between the conjecture and the Brauer-Nesbitt theorem on blocks of defect zero.
The blocks of the periplectic Brauer algebra

SIGISWALD BARBIER, Ghent University, Belgium

In this talk I will introduce the periplectic Brauer algebra $A_n$ using diagrams and determine its blocks in arbitrary characteristic (different from two). We find the surprising result that there is only one block if $n$ is not too small compared to the positive characteristic.

The motivation to study this periplectic Brauer algebra was representation theory of the periplectic Lie superalgebra. They are related via a Schur-Weyl type duality.

Character degrees in $\pi$-separable groups

NICOLA GRIOTTINI, Università degli Studi di Firenze, Italy

The relation between ordinary characters and Brauer characters is one of the main topics in the Theory of Characters of Finite Groups. This is probably why, in 1974, Martin Isaacs studied how to lift the Brauer characters of a $p$-solvable group, i.e., how to find a family of irreducible ordinary characters which coincide with the irreducible Brauer characters if restricted to the $p$-regular elements.

Later, Isaacs decided to find a more general result. In 1982, in the article Character of $\pi$-Separable Groups, using a similar technique he constructed a family of lifts for something like Brauer characters, but generalized for a set of primes instead of just one prime. We will talk about this family of $B_\pi$-characters. After a brief introduction to the theory of $B_\pi$-characters, we will focus on the properties of the degrees of this family of characters $B_\pi(G)$ and on their relation with the group structure.

As expected, in a $\pi$-separable groups the the set of $B_\pi$-character degrees presents some qualities which are already known for the degrees of irreducible Brauer characters. From the set of $B_\pi$-character degrees, in fact, we can tell if the group has a normal $\pi$-complement.

What was not expected is that, if we consider the family $B_\pi(G) \cup B_{\pi'}(G)$, i.e., the family of characters which are either $B_\pi$-characters or $B_{\pi'}$-characters, then the degrees of characters in this family share some properties with the degrees of characters in the whole set $\text{Irr}(G)$. The famous Theorem of Ito-Michler, for example, still holds if we consider the primes which divide not the degrees of some characters in $\text{Irr}(G)$, but just the degrees of some characters in $B_\pi(G) \cup B_{\pi'}(G)$, even if in general this second set is smaller.

Towards the McKay Conjecture and generalizations

BRITTA SPÄTH, Bergische Universität Wuppertal, Germany

The McKay conjecture has become the prototype for global-local conjectures in representation theory of finite groups. We give an overview of how the McKay conjecture has been proven for the prime 2 and what are the missing open problems for checking the conjecture for the remaining primes. We also point out how this influenced various results on similar global-local conjectures.

On the first Zassenhaus conjecture

FLORIAN EISELE, University of Glasgow, UK

There are many interesting problems surrounding the unit group $U(RG)$ of the ring $RG$, where $R$ is a commutative ring and $G$ is a finite group. Of particular interest are the finite subgroups of $U(RG)$. In the seventies, Zassenhaus conjectured that any $u \in U(ZG)$ of finite order is conjugate, in the group $U(QG)$, to an element of the form $\pm g$, where $g$ is an element of the group $G$. This came to be known as the (first) Zassenhaus conjecture. I will talk about the recent construction of a counterexample to this conjecture (this is joint work with L. Margolis), but also on recent work on related questions in the modular representation theory of finite groups.
Combinatorial solutions to the reflection equation
AGATA SMOKTUNOWICZ, University of Edinburgh, UK

In this research we consider applications of noncommutative ring theory in other research areas, namely the Yang-Baxter equation (YBE) and the reflection equation. In 2007 Rump presented some surprising connections between nilpotent rings and set-theoretic solutions of the Yang-Baxter equation. In particular he showed that every nilpotent ring yields a solution to Yang-Baxter equation, and every non-degenerate, involutive set-theoretic solution of Yang-Baxter equation can be obtained from a nilpotent ring, or more generally from a brace. The set-theoretic reflection equation with the first examples of solutions first appeared in the work of Caudrelier and Zhang.

In this talk we use ring-theoretic methods and more generally methods from the theory of braces to produce set-theoretic solutions to the reflection equation. We also use set-theoretic solutions to construct solutions to the parameter dependent reflection equation. Among other things we show that for a finite non-degenerate involutive solution to YBE one only needs to check one of the coordinates to prove that a certain map is a reflection. We also show that every map equivariant under the action of the permutation group $G(X, r)$ yields a reflection.

This talk will be mainly based on a joint work with Leandro Vendramin and Robert Weston.

On characters of degree not divisible by two primes
CAROLINA VALLEJO RODRÍGUEZ, Universidad Autónoma de Madrid, Spain

Let $\pi$ be a set of primes and denote by $\text{Irr}_{\pi'}(G)$ the set of irreducible characters of $G$ of degree coprime to $p$, for all primes in $\pi$. Also, denote by $\text{Lin}(G)$ the set of linear characters of $G$. In the case where $\pi$ consists of a single prime $p$, a celebrated theorem of J. Thompson asserts that, if $\text{Irr}_{\pi'}(G) = \text{Lin}(G)$ then $G$ has a normal $p$-complement. In such a case, the group $G$ is actually solvable. In this talk, I will present a variation on Thompson's result in the case where $\pi$ contains two primes. The key will be that perfect non-trivial groups do not satisfy the character set equality condition. This is joint work with Eugenio Giannelli and Mandi Schaeffer Fry.

The Alperin-McKay conjecture for simple groups of type $A$
JULIAN BROUGH, Bergische Universität Wuppertal, Germany

For a prime $\ell$, the Alperin-McKay conjecture relates the height zero characters of an $\ell$-block of a finite group and its Brauer correspondent. The reduction theorem reduces this conjecture to validating the inductive Alperin-McKay condition for finite quasi-simple groups. In this talk we discuss a new criterion, which has been tailored to blocks for groups of Lie type arising from a reductive connected algebraic group with connected centre. The advantage of this new criterion is that the results from Deligne-Lusztig theory for the representation theory of such groups is then applicable.

In this talk we shall discuss this new criterion and provide an application which validates the inductive Alperin-McKay condition for simple groups of type $SL_n(q)$, when the prime $\ell$ does not divide $6q(q-\epsilon)$ and the block has an abelian stabiliser in the outer automorphism group. This is a joint project with Britta Späth.
Global local conjectures and the Bonnafé–Dat–Rouquier Morita equivalence

LUCAS RUHSTORFER, Bergische Universität Wuppertal, Germany

In recent years, many of the famous global-local conjectures in the representation theory of finite groups have been reduced to the verification of certain stronger conditions on the characters of finite quasi-simple groups. It became apparent that checking these conditions requires a deep understanding of the action of Aut(G) on the characters of a finite simple group G of Lie type.

On the other hand, the Morita equivalence by Bonnafé–Dat–Rouquier has become an indispensable tool to study the representation theory of groups of Lie type. Therefore, it is imperative to understand the action of group automorphisms on characters corresponding to each other under the Bonnafé–Dat–Rouquier Morita equivalence. In this talk, we will discuss recent progress made on this subject.

Groups with few \( p' \)-character degrees

NOELIA RIZO, Universitat de València, Spain

One of the classical problems in character theory of finite groups is what information of a finite group \( G \) can we obtain from its character table. Some of these problems deal with the set of character degrees \( \text{cd}(G) = \{ \chi(1) | \chi \in \text{Irr}(G) \} \) and a given prime number \( p \).

For instance, if we write \( \text{cd}_{p'}(G) = \{ \chi(1) \in \text{cd}(G) | p \nmid \chi(1) \} \), the situations \( \text{cd}_{p'}(G) = \text{cd}(G) \) and \( \text{cd}_{p'}(G) = \{ 1 \} \) have been studied by Itô and Michler ([Ito51], [Mic86]) and Thompson ([Tho70]), respectively. In our work ([GRS19]) we prove a variation of Thompson’s theorem: if \( |\text{cd}_{p'}(G)| = 2 \) then \( G \) is solvable and \( \text{O}^{pp'}(G) \) is trivial. To obtain this result, we need the Classification of Finite Simple Groups.

References


2-categories and 2-representations

VOLODYMYR MAZORCHUK, Uppsala University, Sweden

This will be a survey talk on recent developments in the area of 2-representation theory, that is representation theory of 2-categories, with main emphasis on representations of finitary 2-categories (i.e. 2-analogues of finite dimensional algebras). I will describe several recent results related to classification of various types of 2-representations and also shortly touch upon techniques and methods which are inspired by abstract algebra, classical representation theory and semigroup theory.
**On Brauer trees in blocks of finite groups of Lie type**  
**Radha Kessar**, City, University of London, UK

The context for this talk is the following question: Which Brauer trees occur as $p$-blocks of finite quasisimple groups? Thanks to the miraculous compatibility of Lusztig’s character theory with Brauer’s theory of $p$-blocks and to the cumulative contributions of many mathematicians over the last four decades, a big part of the picture for finite quasisimple groups of Lie type has been filled in. I will report on ongoing work with David Craven on the remaining case, namely the case of non-unipotent blocks.

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**On Lattice-Free Orbit Polytopes**  
**Frieder Ladisch**, University of Rostock, Germany

This talk is about an application of representations of groups to integer convex optimization, and some open questions motivated by this application. Let $G$ be a permutation group, acting on $\mathbb{R}^d$ by permuting coordinates. An integral vector $z \in \mathbb{Z}^d$ is called a core point (for $G$) if the convex hull of its $G$-orbit $Gz$ is lattice-free, that is, contains no other integer vectors except the vectors in the orbit $Gz$ itself. The notion of core points was introduced by Herr, Rehn and Schürmann, motivated by integer convex optimization. An important open problem is to somehow parametrize all core points for a given group. For example, we show that there are only finitely many core points up to a certain equivalence relation when $\mathbb{Q}^d$ modulo the fixed space of $G$ is irreducible. We conjecture that the number of core points up to equivalence is infinite otherwise.

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**Coffee Break**

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**Structure groups and structure racks of YBE solutions**  
**Victoria Lebed**, Université de Caen Normandie, France

Set-theoretic solutions to the Yang-Baxter equation form a vast structure whose current understanding is very limited. Recently two classes of solutions received particular attention: involutive solutions, and solutions coming from self-distributive structures (racks, quandles etc.). Idempotent solutions (in particular those coming from groups) form a third important - and often overlooked! - family. In this talk we will argue that these three families can be seen as three “axes” in the world of YBE solutions. In particular, the structure group and the structure rack of a (nice) solution can be regarded as “projections” onto two of these axes. To illustrate the power of this viewpoint, we will describe a finite quotient of the structure group of a finite solution, which allows one to check important properties of the solution in a finitary setting. The talk will be mainly based on a joint work with L. Vendramin.
Every abstract group has a profinite completion, which equals to the inverse limit of all its finite quotients. Let $G$ be a profinite group, regarding $G$ as an abstract group, it has a profinite completion, $\hat{G}$. A profinite group which is equal to its own profinite completion is called strongly complete. Given a profinite group $G$, we can define the following ascending chain over the ordinals:

$$G_0 = G$$
$$G_{\alpha+1} = \hat{G}_\alpha$$

and for a limit ordinal $\beta$

$$G_\beta = \varprojlim_{\alpha<\beta} G_\alpha.$$ 

Obviously, if $G$ is strongly complete, then $G_\alpha = G$ for all $\alpha$. We show that for a nonstrongly complete profinite group, the chain of profinite completions never terminates. Moreover, every group in the chain is a proper subgroup of all the following groups.

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**Special Classes of Homogeneous Semilocal Rings: Corner Rings**

SUSAN F. ELDENKEN, Helwan University, Egypt

In this article we are interested to find to what extent the ring-theoretic properties of homogeneous semilocal rings are preserved by its corner ring $eRe$, where $e \in \text{Id}(R)$, and visa versa. The main result of this work is to show that if $R$ is a ring with an arbitrary idempotent $e$ such that both $eRe$ and $(1-e)R(1-e)$ are homogeneous semilocal rings, then $R$ need not be a homogeneous semilocal ring. In particular, under certain additional circumstances, $R$ is homogeneous semilocal ring. Moreover, there are many results concerning semi-boolean, nil-clean and abelian property.

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**New Types of Permuting $n$-Derivations of Semiprime Rings**

MEHSIN JABEL ATTEYA, University of Leicester, UK and Al-Mustansiriyah University, Iraq

In this article we introduce the concepts of a permuting $n$-semiderivation, a permuting $n$-generalized $n$-semiderivation and a permuting $n$-semigeneralized $n$-semiderivation of semiprime ring $R$. The article divided into four sections where in each section we focus on one type of previous definitions.

In fact, in (2018) the authors Ajla Fošner and Mehsin Jabel Atteya [1] introduced the dentition of semigeneralized semiderivation of a ring $R$ with some results about it as follow. An additive mapping $D : R \rightarrow R$ is called a semigeneralized semiderivation associated with a semiderivation $d : R \rightarrow R$ and the functions $h, g : R \rightarrow R$ if (i) $D(xy) = D(x)h(y) + g(x)d(y)$, (ii) $D(xy) = D(xy) + h(x)(D(y)) + (ii)D(d(x)) = d(D(x))$, (iii) $D(g(x)) = g(D(x))$, (iv) $D(h(x)) = h(D(x))$ and (v)$g(h(x)) = h(g(x))$ for all $x, y \in R$.

More precisely, in (1983) the author J.Bergen [2] has introduced the notation of semiderivations of a ring $R$ which extends the notation of derivation of a ring $R$, as following: $f : R \rightarrow R$ is a semiderivation of $R$ if there exists a function $g : R \rightarrow R$ such that $(i)f(xy) = f(x)g(y) + xf(y) = f(x)y + g(x)f(y)$ for all $x, y \in R$ and (ii)$f(g(x)) = g(f(x))$ for all $x \in R$.

Throughout $R$ will represent an associative ring with center $Z(R)$. A ring $R$ is said to be prime if $xRy = 0$ implies that either $x = 0$ or $y = 0$ and semiprime if $xRx = 0$ implies that $x = 0$; where $x, y \in R$. A prime ring is obviously semiprime. For any $x, y \in R$, the symbol $[x, y]$ stands for the commutator $xy - yx$ and the symbol $(x,y)$ stands for the commutator $xy + yx$.

Also, $R$ is said to be an $n$-torsion free, where $n \neq 0$ is an integer, if whenever $nx = 0$, with $x \in R$ then $x = 0$. An additive mapping $d : R \rightarrow R$ is called a derivation if $d(xy) = d(x)y + xd(y)$ holds for all $x, y \in R$. Let $S$ be a nonempty subset of $R$. A mapping from $R$ to $R$ is called centralizing on $S$ if $[d(x), x] \in Z(R)$ for all $x \in S$ and is called commuting on $S$ if $[d(x), x] = 0$; for all $x \in S$. Some results:

Theorem. Let $R$ be a semiprime ring and $\Delta$ is a non-zero permuting $n$-semiderivation of $R$ with a trace $\delta$ such that $\Delta$ acts as a right-multiplier. If $R$ admits to satisfy the identity $[\Delta(R_1), \Delta(R_2)] \in Z(R)$. Then (i) $R$ has a weak zero-divisor. (ii) either $\Delta^2 \in AM$-set or if $\delta$ acts as a surjective function then $\delta^2 \in Z(R)$.
References


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**Structure and properties of Hecke–Kiselman monoids and algebras**

Magdalena Wiertel, University of Warsaw, Poland

To every finite graph $\theta$ with $n$ vertices one can associate a finitely presented monoid $HK_{\theta}$, called the Hecke–Kiselman monoid. It is a monoid generated by $n$ idempotents with relations of the form $xy = yx$, $xy = yxy$ or $xy = yyx = xy$, depending on the edges between vertices $x$ and $y$ in $\theta$. We focus on the structure of Hecke–Kiselman monoids and the associated semigroup algebras in the case of directed graphs.

We investigate the interplay between combinatorial and structural properties. In the case $\theta$ is a cycle of length $n \geq 3$, a hierarchy of certain structures of matrix type is discovered within the monoid $C_n = HK_{\theta}$. This allows us to describe the ring theoretical properties of the algebra $K[C_n]$ and apply the obtained results to the case of arbitrary directed graphs. In particular, for such graphs, we characterize all Noetherian Hecke–Kiselman algebras $K[HK_{\theta}]$ in terms of the graph $\theta$.

The talk is based on a joint work with J. Okniñski.

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**Lunch**

12:45

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**Some properties of the conjugacy class sizes of a finite group**

Silvio Dolfi, University of Firenze, Italy

Let $G$ be a finite group and $\text{cs}(G)$ the set consisting of the sizes of the conjugacy classes of $G$. Addressing the question of the relations between the set $\text{cs}(G)$ and the structure of the group $G$, I will first review some classical as well as some more recent results. Next, I will talk about a graph related to the set $\text{cs}(G)$. The *prime graph* on class sizes $\Delta(G)$ is defined as the simple undirected graph having the primes that divide some $n \in \text{cs}(G)$ as vertices, and as edges the pairs of distinct primes $(p, q)$ such that the product $pq$ divides some $n \in \text{cs}(G)$. I will discuss a few properties of the graph $\Delta(G)$ and, in particular, the existence of some large complete subgraphs. Finally, I will mention the connections with a similar graph related to the degrees of the irreducible characters of a finite group.
14:30  On the structure monoid and algebra of left non-degenerate set-theoretic solutions to the Yang–Baxter equation
Arne Van Antwerpen, Vrije Universiteit Brussel, Belgium

Let $r : X^2 \to X^2$ be a set-theoretic solution of the Yang–Baxter equation on a finite set $X$. Denote $M(X, r)$ for the structure monoid $\langle x \in X \mid xy = uv \text{ if } r(x, y) = (u, v) \rangle$. For a finite involutive non-degenerate solution $(X, r)$ of the Yang–Baxter equation it is shown by Gateva-Ivanova and Van den Bergh that the structure monoid $M(X, r)$ is a monoid of I-type, and the structure algebra $K[M(X, r)]$ over a field $K$ shares many properties with commutative polynomial algebras, in particular, it is a Noetherian PI-domain that has finite Gelfand–Kirillov dimension. Motivated by recent work of Lebed and Vendramin for non-degenerate set-theoretic solutions and some results of Jespers and myself for solutions associated to semi-braces, we will discuss the structure monoid for bijective left non-degenerate set-theoretic solutions and its associated algebra over a field $K$. Using a realization of Lebed and Vendramin of $M(X, r)$ as a regular submonoid in the semidirect product $A(X, r) \rtimes \text{Sym}(X)$, where $A(X, r)$ is the structure monoid of the derived solution associated to $(X, r)$, we will show that $K[M(X, r)]$ is a finite module over a central affine subalgebra. In particular, $K[M(X, r)]$ is a Noetherian PI-algebra of finite Gelfand–Kirillov dimension bounded by $|X|$.

Moreover, we will characterize, in ring-theoretical terms of $K[M(X, r)]$, when $(X, r)$ is an involutive solution. This characterization provides, in particular, a positive answer to a recent conjecture of Gateva-Ivanova concerning the cancellativity of $M(X, r)$.

We will relate the prime spectra of the monoids $M(X, r)$ and $A(X, r)$ and show that for bijective square-free left non-degenerate solutions a complete description can be given. Moreover, we will discuss that prime ideals of $M(X, r)$ are determined by the divisibility structure of $M(X, r)$. As the apotheosis of this talk, we will use these results to give a description of the prime spectrum of the structure algebra $K[M(X, r)]$.

Joint work w. E. Jespers and L. Kubat.

15:00  Coffee Break

15:30  On structure algebras of idempotent solutions of the Yang–Baxter equation
Lukasz Kubat, Vrije Universiteit Brussel, Belgium

In this short talk I will focus on recent results concerning ring-theoretical properties of the monoid algebra $K[M(X, r)]$ over a field $K$ of the structure monoid $M(X, r) = \langle X \mid xy = uv \text{ whenever } r(x, y) = (u, v) \rangle$ of an idempotent solution $(X, r)$ of the Yang–Baxter equation.

This is a joint work with Eric Jespers, Arne Van Antwerpen and Charlotte Verwimp.

15:45  Skeleton groups and their isomorphism problem
Subhrajyoti Saha, Monash University, Australia

Classification of groups up to isomorphism is one of the main themes in group theory. A particular challenge is to understand finite $p$-groups, that is, groups of $p$-power order for a prime $p$. Since a classification of $p$-groups by order seems out of reach for large exponents $n$, other invariants of groups have been used to attempt a classification; a particularly intriguing invariant is coclass. A finite $p$-group of order $p^n$ and nilpotency class $c$ has coclass $r = n - c$.

Recent work in coclass theory is often concerned with the study of the coclass graph $G(p, r)$ associated with the finite $p$-groups of coclass $r$. The vertices of the coclass graph $G(p, r)$ are (isomorphism type representatives of) the finite $p$-groups of coclass $r$, and there is an edge $G \to H$ if and only if $G$ is isomorphic to $H/\gamma(H)$ where $\gamma(H)$ is the last non-trivial term of the lower central series of $H$. It is known that one of the feasible approaches for investigating $G(p, r)$ is to first focus on so-called skeleton groups.
Our definition of skeleton groups for the graph $G(p,r)$ is based on the technical definition of ‘constructible groups’ given by Leedham-Green. For odd $p$, let $S$ be an infinite pro-$p$ group and informally skeleton groups can be described as twisted finite quotients of $S$, where the twisting is induced by some suitable homomorphism. Understanding these groups is a crucial step towards investigating the structure of $G(p,r)$. The first major step towards this is to investigate when two skeleton groups are isomorphic. We present some general results towards this direction including two interesting special cases.

This is a joint work with Dr. Heiko Dietrich, Monash University, Australia.

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**Prime monomial ideals of subsemigroup algebras of free nilpotent groups**

Tomer Bauer and Be’eri Greenfeld, Bar-Ilan University, Israel

We study prime monomial homomorphic images of the semigroup algebra of the subsemigroup of the 2-generated free 3-nilpotent group generated by the (positive) group generators. In particular, we show that they are either PI of linear growth or primitive just-infinite of quadratic growth. This partially answers a question of Okniński.

This is a joint work with Be’eri Greenfeld.

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**On the Zassenhaus Conjecture for certain cyclic-by-nilpotent groups**

Mauricio Caicedo, Vrije Universiteit Brussel, Belgium

Hans Zassenhaus conjectured that every torsion unit of the integral group ring of a finite group $G$ is conjugate within the rational group algebra to an element of the form $\pm g$ with $g \in G$. This conjecture has been disproved recently for metabelian groups, by Eisele and Margolis. However it is known to hold for many classes of solvable groups, as for example nilpotent groups, cyclic-by-abelian groups and groups having a Sylow subgroup with abelian complement. On the other hand, the conjecture remains open for the class of supersolvable groups. In this talk we focus on this question. More precisely, we focus on the study of the Zassenhaus Conjecture for the class of cyclic-by-nilpotent groups with special attention to the class of cyclic-by-Hamiltonian groups. This is a joint work with Prof. Angel del Río.

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**Structure groups**

Wolfgang Rump, University of Stuttgart, Germany

There is a particular class of groups which can be associated to certain mathematical structures in a specific way, providing a group invariant to the given structure. Structures to which such a group can be attached, arise in general and algebraic topology, algebraic logic, number theory, functional analysis and measure theory, non-commutative geometry and operator theory. A typical example is the knot group associated to a given knot or link. In this and many other cases, the structure group is lattice ordered, and its construction goes through an intermediate step: an L-algebra. We explain the relationship between L-algebras and their structure groups and illustrate the general pattern by an example of Euclidean geometry, a particular case of a wide class of L-algebras where the structure group is a classifying invariant.
Perpetuants: a lost treasure
CLAUDIO PROCESI, Università degli Studi di Roma, Italy

Perpetuant is one of the several concepts invented (in 1882) by J. J. Sylvester in his investigations of covariants for binary forms. It appears in one of the first issues of the American Journal of Mathematics which he had founded a few years before. It is a name which will hardly appear in a mathematical paper of the last 70 years, due to the complex history of invariant theory which was at some time declared dead only to resurrect several decades later. I learned of this word from Gian-Carlo Rota who pronounced it with an enigmatic smile.

In this talk I want to explain the concept, a Theorem of Stroh, and some new explicit description.

McKay correspondences of Brauer characters
JOAN F. TENT, University of Valencia, Spain

Let $G$ be a finite group, let $p$ be a prime and let $P \in \text{Syl}_p(G)$. Suppose that $q \neq p$ is another prime, let $\text{IBr}(G) := \text{IBr}_q(G)$ be the set of irreducible $q$-Brauer characters of $G$, and denote by $\text{IBr}_p'$ the set of irreducible $q$-Brauer characters of $G$ with degree coprime to $p$. We recall that it is not true in general that $|\text{IBr}_p'(G)| = |\text{IBr}_p'(N_G(P))|$, even if we assume that $G$ is $p$-solvable. On the other hand, if $G$ is $q$-solvable and the non-abelian simple groups of order divisible by $p$ involved in $G$ satisfy the inductive McKay conjecture, then by [2] we do have equality $|\text{IBr}_p(G)| = |\text{IBr}_p(N_G(P))|$.

Our aim in this talk is to present the following result:

**Theorem.** Let $G$ be a finite group and let $P \in \text{Syl}_p(G)$, where $p \geq 5$. Suppose that $N_G(P) = PQ$, where $Q$ is a $q$-group with $q \neq p$, and assume that $G$ is $q$-solvable. If $\chi \in \text{IBr}_p'(G)$ then

$$\chi_{N_G(P)} = \mu + \Delta,$$

where $\mu \in \text{IBr}_p'(N_G(P))$ and $\Delta$ is a possibly zero $q$-Brauer character all whose irreducible constituents have degree divisible by $p$. Furthermore, the map $\chi \mapsto \mu$ is a bijection

$$\text{IBr}_p'(G) \longrightarrow \text{IBr}_p'(N_G(P)),$$

where $\chi \in \text{IBr}_p'(G)$ and $\mu \in \text{IBr}_p'(N_G(P))$ is a constituent of $\chi_{N_G(P)}$.

Note that if $q \nmid |G|$ in the previous theorem, then $N_G(P) = P$ and the result in this case is implied by Corollary B of [1].

**References**


Irrationality of quotient varieties
URBAN JEZERNIK, University of the Basque Country, Spain

The rationality problem in algebraic geometry asks whether a given variety is birational to a projective space. We will focus on varieties that arise from representations of finite groups, principally in the direction of constructing irrational examples via an attractive cohomological obstruction.
Central Normalized Torsion Units of Integral RBAs and Group Rings
Gurmiah Singh, University of Regina, Canada

A reality-based algebra (RBA) is a finite-dimensional associative algebra with involution that has a distinguished basis $B$ which contains $1_A$ and is closed under a pseudo-inverse condition. If the RBA has a one-dimensional representation taking positive values on $B$, then we say that the RBA has a positive degree map. When the structure constants relative to a standardized basis of an RBA with positive degree map are all integers, we say that the RBA is integral. Integral RBAs are closely related to group rings of finite groups. In this talk we will prove that the central torsion units of integral RBAs with positive degree map are trivial. As a consequence of this result, normalized torsion units of integral group rings need not be central in the whole group ring but only central in a nice sub-RBA of the group ring to become trivial. This is joint work with Allen Herman.

The Multiplicative Jordan Decomposition in the Integral Group Ring $\mathbb{Z}[Q_8 \times C_p]$
Wei-Liang Sun, National Taiwan Normal University, Taiwan

Let $G$ be a finite group and $\mathbb{Q}[G]$ be a rational group algebra. Then every invertible element $\alpha$ of $\mathbb{Q}[G]$ has a unique multiplicative Jordan decomposition $\alpha = \alpha_s \alpha_u$ where $\alpha_s$ is a semisimple unit, $\alpha_u$ is a unipotent unit and $\alpha_s \alpha_u = \alpha_u \alpha_s$. Following A. W. Hales and I. B. S. Passi, we say that $G$ has the multiplicative Jordan decomposition property (MJD) if for every unit $\alpha$ in the integral group ring $\mathbb{Z}[G]$, both $\alpha_s$ and $\alpha_u$ are contained in $\mathbb{Z}[G]$. It is a challenging problem to determine which groups have MJD since 1991. However, there is no answer whether the MJD property holds for groups $Q_8 \times C_p$ where $p > 5$ is prime such that $2$ has even multiplicative order modulo $p$. In this talk, I will briefly present the history of MJD problem and give a new result that $Q_8 \times C_p$ has MJD for infinitely many $p$.

This is a joint work with Wentang Kuo (University of Waterloo, Canada).

A note on normal complement problem for split metacyclic groups
Surinder Kaur, Indian Institute of Technology Ropar, India

In this talk, we discuss the normal complement problem for metacyclic groups in modular group algebras. If $F$ is the field with $p$ elements and $G$ is a finite split metacyclic $p$-group of nilpotency class 2, then we prove that $G$ has a normal complement in the unit group $U(FG)$ of $FG$. For a finite field $F$ of characteristic $p$, where $p$ is an odd prime, we prove that the dihedral group $D_{2p^{m}}$ has a normal complement in $U(FD_{2p^{m}})$ if and only if $p = 3$ and $|F| = 3$.

References
1. A note on normal complement problem for split metacyclic groups (to appear in Communications in algebra)

Local properties of cut groups
Andreas Bächle, Vrije Universiteit Brussel, Belgium

A cut group is a finite group $G$, such that $\mathbb{Z}(U(ZG)) = \pm \mathbb{Z}(G)$, i.e. the central units of its integral group ring are as sparse as possible. This class of groups was also independently studied out of purely group theoretic (inverse semi-rational groups) and character theoretic interest. We will briefly discuss these connections, some recent results and compare them to corresponding classical results on the subclass of rational groups. This will lead us to propose some open questions on their $p$-subgroups. This is based on joint work with M. Caicedo, E. Jespers and S. Maheshwary.

References
12:45 Lunch

14:00 Generalisations of Weiss
Pavel Zalesskii, Universidade de Brasilia, Brazil

We shall discuss generalisations of the classical Theorem of Alfred Weiss (1988) on integral $p$-adic representations of a finite $p$-group.

14:30 Simple and projective representations of finite sets
Jacques Thévenaz, École Polytechnique Fédérale de Lausanne, Switzerland

A correspondence between two finite sets is a subset of their direct product. Correspondences can be composed and this gives rise to the category of finite sets and correspondences. In a joint work with Serge Bouc, we study the linear representations of this category, which turn out to have many remarkable features. In particular, we shall explain how the simple representations are parametrized in terms of finite posets. We shall sketch how to solve the difficult problem of finding the dimension of every evaluation of such a simple representation. Finally, we shall describe precisely all the posets for which the associated simple representations are projective.

15:00 Coffee Break

15:30 Fixed points of automorphisms of free-abelian times free groups
Mallika Roy, BGSMath-UPC, Barcelona, Spain

The behavior of the automorphisms and the properties of auto-fixed subgroups of free groups $F_n$ is well known and very studied in the literature. On the other hand the automorphisms of $\mathbb{Z}^m$ are just matrices from $GL(m, \mathbb{Z})$. But fixed point subgroups of $\mathbb{Z}^m \times F_n$ are much more complicated and have more degenerated behaviour. In the talk I will mainly focus on the differences between automorphisms and fixed points of $F_n$ and of $\mathbb{Z}^m \times F_n$. I will present some results about ranks of fixed subgroups and a periodicity formula for the automorphisms of $\mathbb{Z}^m \times F_n$ and algorithmic computation of the auto-closure of a given subgroup in $\mathbb{Z}^m \times F_n$. All together it is related with the concept of "factor" in this family of groups.

This is a joint work with E. Ventura.

15:45 On the rationality of growth series for Artin Groups
Islam Foniqi, University of Milano-Bicocca, Italy

Artin Groups constitute a broad and mysterious class of finitely presented groups. We define them by a Coxeter System. Then we formulate spherical and geodesic growth (series) with respect to the natural presentation. From the combinatorial point of view it is important to decide if the growth series are rational. We will present some of the results already achieved towards this objective, and discuss how these connect to geometry, and regular languages.
From the YBE to the Left Braces

IVAN LAU, University of Edinburgh, UK

In this talk, we introduce the Yang-Baxter equation (YBE) and its solutions. We then introduce left braces and illustrate how this algebraic structure helps to study the solution of the YBE. Beyond its physical significance, we will also discuss why the algebraic properties of left braces is of great mathematical interest. In particular, we will show that a left brace with an associative operation $*$ is a ring. This is joint work with Patrick Kinnear and Dora Puljić.

On homomorphisms of crossed products of locally indicable groups to division algebras

ANDREI JAIKIN-ZAIPIRAIN, Universidad Autonoma de Madrid, Spain

Let $E \ast G$ be a crossed product of a division algebra $E$ and a group $G$. We discuss several problems concerning division $E \ast G$-algebras such as the Macertificateslcev problem on embedding of crossed products $E \ast G$ into division rings, the Atiyah problem about the values of von Neumann dimensions of finitely presented $\mathbb{C}[G]$-modules and the problem of the existence of universal (in the sense of P. Cohn) division $E \ast G$-algebra.

In particular, we will explain how $E \ast G$ can be embedded in a division algebra if $G$ is locally indicable. Moreover, we will show that this division algebra is Hughes-free and in many cases it is also universal.
Posters

Quasi-linear left cycle sets and the Retraction Problem
MARCO CASTELLI and GIUSEPPINA PINTO, Università del Salento, Italy

In 2014, Rump introduced the algebraic structure of quasi-linear left cycle sets, a useful framework to work with the set-theoretic solutions of the Yang-Baxter equation. A left cycle set \((A, \cdot)\) is called \textit{quasi-linear} if \(A\) is endowed with an additive abelian group structure such that
\[
a \cdot (b + c) = a \cdot b + (a - b) \cdot c
\]
for all \(a, b, c \in A\). For a quasi-linear left cycle set \(A\), Rump introduced two ideals, the \textit{Socle} and the \textit{Radical} of \(A\), indicated by \(\text{Soc}(A)\) and \(\text{Rad}(A)\) respectively, and another classical tool, the \textit{Fixator} of \(A\), denoted by \(\text{Fix}(A)\). In [1] we show a connection between the quasi-linear left cycle sets with \(\text{Rad}(A) \subseteq \text{Soc}(A)\) and the \textit{unitary metahomomorphisms} on abelian groups, introduced by Gu (1997) in order to find new solutions of the Yang-Baxter equation.

In this poster, we give a characterization of the quasi-linear left cycle sets \(A\) with \(\text{Rad}(A) \subseteq \text{Soc}(A)\) via unitary metahomomorphisms and we give a complete description of those for which \(\text{Rad}(A) \subseteq \text{Fix}(A) = \text{Soc}(A)\) improving the results obtained by Catino and Miccoli (2015). Moreover, we consider a problem posed by Rump (2014) involving the Retract Relation and the related partial results obtained in [1].

References


Proving six cases of the BMM trace conjecture through computational methods
EIRINI CHAVLI, University of Stuttgart, Germany

Two decades years ago, Broué, Malle and Rouquier published a paper in which they associated to every complex reflection group two objects which were classically associated to real reflection groups: a braid group and a Hecke algebra. Their work was further motivated by the theory, developed together with Michel, of “Spetses”, which are objects that generalise finite reductive groups in the sense that their associated Weyl groups are complex reflection groups. The four of them advocated that several nice properties of braid groups and Hecke algebras generalise from the real to the complex case, culminating in two main conjectures as far as the Hecke algebras are concerned: the “BMR freeness conjecture” and the “BMM trace conjecture”. The two conjectures are the cornerstones in the study of several subjects that have flourished in the past twenty years, but had remained open until recently for the exceptional complex reflection groups. Recently, the proof of the “BMR freeness conjecture” was completed for all exceptional complex reflection groups. In this poster, we present our proof of the “BMM trace conjecture” for six exceptional groups. This is joint work with Christina Boura, Maria Chlouveraki and Konstantinos Karvounis.

The solutions of the Yang-Baxter equation with finite order
ILARIA COLAZZO, Università del Salento, Italy

The Yang-Baxter equation is a fundamental tool in several different fields of research such as statistical mechanics, quantum group theory, and low-dimensional topology. In this poster, we focus on the set-theoretical solutions of the Yang-Baxter equation which are of finite order and not necessarily bijective. We use the matched product of solutions as a unifying tool for treating these solutions of finite order, that also include involutive and idempotent solutions. In particular, we show that the matched product of two solutions \(r_S\) and \(r_T\) is of finite order if and only if \(r_S\) and \(r_T\) are. Moreover, we analyse the associated solution associated with a semi-brace.
Quasigroups and biracks
Přemysl Jedlička, Czech University of Life Sciences, Czech Republic
and Agata Pilitowska, Anna Zamojska-Dzienio, Warsaw University of Technology, Poland

It is known (see e.g. [5, 2, 1]) that there is a one-to-one correspondence between non-degenerate (involutive) solutions of the Yang-Baxter equation and (involutive) biracks \((X, \circ, \setminus, \cdot, /, \cdot, /)\) – algebras which have a structure of two one-sided quasigroups \((X, \circ, \setminus)\) and \((X, \cdot, /)\) and satisfy some additional conditions. This fact allows one to characterize non-degenerate solutions of the Yang-Baxter equation applying the universal algebra tools.

We present quasigroups, one-sided quasigroups and biracks and show how to apply them to construct all involutive solutions of multipermutation level 2. Such solutions fall into two disjoint classes – distributive ones and non-distributive ones. Using this construction, we enumerate all distributive involutive solutions up to size 14.

References

Solutions to the reflection equation from the theory of braces
Kyriakos Katsamaktsis, University of Edinburgh, UK

Building on the recent preprint [1] of Smoktunowicz, Vendramin and Weston, we obtain solutions to the reflection equation using braces. In particular we show that every brace yields solutions to the reflection equation, some of which are involutive, thus giving solutions to the parameter-dependent reflection equation. We also find reflections coming from factorizable rings by adapting results from [1].

References

Morphic elements in regular near-rings
Ivan Philly Kimuli, Alex Samuel Bamunoba and David Ssevviiri, Makerere University, Uganda

We define morphic near-ring elements and study their behavior in regular near-rings. We show that the class of left morphic regular near-rings is properly contained between the classes of left strongly regular and unit regular near-rings.
New set-theoretical solutions of the pentagon equation
MARZIA MAZZOTTÀ AND PAOLA STEFANELLI, Università del Salento, Italy

The pentagon equation is profusely investigated in the modern mathematical physics and it plays an essential role in the development of harmonic analysis on quantum groups. Moreover, it appears in various contexts and with different terminologies, for instance, if \( \mathcal{H} \) is a Hilbert space, a unitary operator from \( \mathcal{H} \otimes \mathcal{H} \) into itself is said to be multiplicative if it satisfies the pentagon equation.

Our attention is posed on the study of set-theoretical solutions on a set \( M \), that are maps \( s : M \times M \to M \times M \) satisfying the relation

\[
s_{23} s_{13} s_{12} = s_{12} s_{23},
\]

where \( s_{12} = s \times id_M \), \( s_{23} = id_M \times s \) and \( s_{13} = (id_M \times \tau)(s_{12}(id_M \times \tau)) \) with \( \tau \) the flip map on \( M \times M \) given by \( \tau(x, y) = (y, x) \). First examples of solutions have been provided in the pioneering works by Zakrzewski (1992), Baaj and Skandalis (2003), and Kashaev and Reshetikhin (2007).

In this poster, we show the complete description of set-theoretical solutions of the form

\[
s(x, y) = (x \cdot y, \theta_x(y))\]

where \((M, \cdot)\) is a group and \( \theta_x \) is a map from \( M \) into itself, for every \( x \in M \), contained in [1]. Moreover, we introduce a new technique to construct set-theoretical solutions of the famous quantum Yang-Baxter equation by using solutions of the pentagon equation.

References


Character triples and group graded equivalences
VIRGILIUS-AURELIAN MINUȚĂ, Babes-Bolyai University, Romania

Let \( N \) be a normal subgroup of \( G \), \( G' \) a subgroup of \( G \), and \( N' \) a normal subgroup of \( G' \). We assume that \( N' = G' \cap N \) and \( G = G'N \), hence \( G := G/N \cong G'/N' \). Let \( b \in Z(ON) \) and \( b' \in Z(ON') \) be \( G \)-invariant block idempotents. We denote \( A := bOG \) and \( A' := b'OG' \). Then \( A \) and \( A' \) are strongly \( G \)-graded algebras, with 1-components \( B \) and \( B' \) respectively. Additionally, assume that \( C_G(N) \subseteq G' \), and denote \( C := OC_G(N) \), which is regarded as a \( G \)-graded \( G \)-acted algebra.

In [2, Definition 2.7.], Britta Späth considers a relation \( \geq_c \) between the character triples \((G, N, \theta)\) and \((G', N', \theta')\), where \( \theta \) is \( G \)-invariant irreducible character belonging to the block \( b \) and \( \theta' \) is a \( G \)-invariant irreducible character belonging to the block \( b' \).

We introduce \( G \)-graded \((A, A')\)-bimodules over \( C \) and we study Morita equivalences between \( A \) and \( A' \) induced by such bimodules.

We prove that if \( \theta \) corresponds to \( \theta' \) under a \( G \)-graded Morita equivalence over \( C \), then \((G, N, \theta) \geq_c (G', N', \theta')\).

We also show that an analogue of the so-called “butterfly theorem” [2, Theorem 2.16] holds for \( G \)-graded Morita equivalences over \( C \).

References

On primitive idempotents of semisimple group algebras
Gabriela Olteanu, Babes-Bolyai University, Romania

We present two methods to construct complete sets of orthogonal primitive idempotents of simple components of semisimple group algebras over finite Galois extensions of the rationals and some classes of finite groups, including finite nilpotent groups.
This is joint work with Inneke Van Gelder.

The Ring of Hybrid Numbers and Some Applications to Linear Algebra
Mustafa Özdemir, Akdeniz University, Turkey

Hybrid numbers are a ring of numbers that unify the field of the complex number \( \mathbb{C} \) and, the rings of hyperbolic numbers \( (\mathbb{H} = \{a + \textbf{h}b : \textbf{h}^2 = 1\}) \) and dual numbers \( (\mathbb{D} = \{a + \varepsilon b : \varepsilon^2 = 0\}) \). In this study, hybrid numbers and some algebraic properties will be introduced briefly and some linear algebra applications will be given by the help of ring homomorphism between hybrid numbers and \( 2 \times 2 \) matrices.

References

The locally nil radical for modules over commutative rings
David Ssevviiri and Annet Kyomuhangi, Makerere University, Uganda

Let \( R \) be a commutative unital ring and \( a \in R \). We introduce and study properties of a functor \( a \Gamma_a(-) \), called the locally nil radical on the category of \( R \)-modules. \( a \Gamma_a(-) \) is a generalisation of both the torsion functor (also called section functor) and Baers lower nil radical for modules. Several local-global properties of the functor \( a \Gamma_a(-) \) are established. As an application, results about reduced \( R \)-modules are obtained and hitherto unknown ring theoretic radicals as well as structural theorems are deduced. For instance, if \( M \) is a reduced \( R \)-module, then the local cohomology functor \( H^i_a(M) \) coincides with \( \text{Ext}_a^i(R/a, M) \); and for any ring \( R \), \( a \Gamma_a(R)[x] = a \Gamma_a(R[x]) \).
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